



SEM-III

GOVERNMENT OF ANDHRA PRADESH

LIFE SKILL COURSE

Second Year B.A., B.Com., B.Sc.,

ANLYTICAL SKILLS

Energised Text Book



SOCIETY OF JKC
COMMISSIONERATE OF COLLEGIATE EDUCATION
ANDHRA PRADESH

ANALYTICAL SKILLS

SEMESTER – III

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First Edition: 2022

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Foreword

Analytical Skills, a Life Skill Course, is offered in the third semester of the second-year undergraduate study. **Analytical skill** is the ability to deconstruct information into smaller categories in order to draw conclusions.^[1] Analytical skill consists of categories that include logical reasoning, critical thinking, communication, research, data analysis and creativity. Analytical skill is taught in contemporary education with the intention of fostering the appropriate practices for future professions. The professions that adopt analytical skill include educational institutions, public institutions, community organizations and industry.



Thinking analytically is a skill that everyone should embark. It can be taught, it can be learned, and it can improve with practice. But like many other skills, such as riding a bike, it is not learned by sitting in a classroom and being told how to do it. Analysts learn by doing.

Choice Based Credit System (CBCS) in all colleges across all undergraduate courses in Andhra Pradesh as per the revised curriculum framework:2020. CBCS has brought a paradigm shift from teacher centered learning approach to student centric learning which offers more flexibility to the student in choosing interdisciplinary, intra-disciplinary and skill-based courses depending on his interest. In brief, it offers opportunities and avenues to learn core subjects but also

allows for exploring additional avenues of learning beyond the core subjects for holistic development.

Such additional avenues of learning are covered under non-core subjects as two skill sets of courses. They are

I. Life Skill Courses

II. Skill Development Courses

The LSCs and SDCs are offered in the first three semesters. As there are no specified Text Books for these courses, the Department of Collegiate Education has come up with the initiative of “Content Generation for Skill Courses” in a phased manner. In the second phase, the content is proposed to be prepared for the benefit of students in their third semester. Accordingly, it was decided to bring out a textbook, prepare e-content in four quadrants (video, text, self-assessment & additional information) along with a podcast in the assigned topic.

I congratulate & appreciate the content developers for bringing a concise yet resourceful material for the benefit of UG students that shall make them more knowledgeable and employable.



Dr. Pola Bhaskar, IAS

Commissioner of Collegiate Education

CBCS/ SEMESTER SYSTEM
(w.e.f. 2022-23)

ANALYTICAL SKILLS

Syllabus

Total 30 Hrs

Course Objective: Intended to inculcate quantitative analytical skills and reasoning as an inherent ability in students.

UNIT – 1: (10 Hours)

Arithmetic ability: Algebraic operations BODMAS, Fractions, Divisibility rules, LCM & GCD (HCF).

Verbal Reasoning: Number Series, Coding & Decoding, Blood relationship, Clocks, Calendars.

UNIT – 2: (10 Hours)

Quantitative aptitude: Averages, Ratio and proportion, Problems on ages, Time-distance – speed.

Business computations: Percentages, Profit & loss, Partnership, simple compound interest.

UNIT – 3: (07 Hours)

Data Interpretation: Tabulation, Bar Graphs, Pie Charts, line Graphs. Venn diagrams.

Recommended Co-Curricular Activities (03 hrs)

Surprise tests / Viva-Voice / Problem solving/Group discussion.

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	1.8	Blood Relationship	
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UNIT-I

ARITHMETIC ABILITY

Algebraic Operations -BODMAS
Fractions
Divisibility Rules
LCM
GCD (HCF)

&

VERBAL REASONING

Number Series
Coding & Decoding
Blood Relationship
Clocks
Calendars

1.1 Algebraic Operations - BODMAS

LEARNING OUT COMES:

After completion of this topic, students will be able to understand.

1. Meaning of BODMAS
2. Concept of BODMAS
3. Simple methods using BODMAS Rule
4. Correct method to solve given problems in algebraic operations.

INTRODUCTION:

To solve any arithmetic expression which involves different algebraic operations like brackets, additions, subtractions, multiplications and divisions, it requires a particular sequence of operations. The Sequence of mathematical operations is BODMAS.

What is BODMAS ?

To solve an arithmetic expression correctly, we have to follow the `order of mathematical operations appear in the letters of the word BODAMS.

B stands for bracket _____ , () , { } , []

O stands for of (or) order

D stands for Division $\div$

M stands for multiplication $\times$

A stands for addition $+$

S stands for subtraction $-$

Example:

1. Simplify $-\overline{2-5} = -(-3) = 3$

2. Simplify $6-[4+\{6-(5-\overline{2-6})\}]$

Answer :

$$\begin{aligned} & 6-[4+\{6-(5-\overline{2-6})\}] \\ &= 6-[4+\{6-(5-(-4))\}] \\ &= 6-[4+\{6-(5+4)\}] \\ &= 6-[4+\{6-9\}] \\ &= 6-[4+(-3)] \\ &= 6-1 = 5 \end{aligned}$$

3. Simplify

$$5 \div [3(6-4)]$$

$$\text{Answer : } 5 \div [3 (6-4)]$$

$$= 5 \div [3(2)]$$

$$= 5 \div 6 = \frac{5}{6}$$

Algebraic Formulae for Utilization

1 (i) $(a + b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $(a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$

(iv) $(a + b)^2 - (a - b)^2 = 4ab$

(v) $(a+b)(a-b) = a^2 - b^2$

2. (i) $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

(ii) $(a - b)^3 = a^3 - 3a^2b + 3ab^2 - b^3$

(iii) $(a + b)^3 + (a - b)^3 = 2(a^3 + 3ab^2)$

(iv) $(a + b)^3 - (a - b)^3 = 2(3a^2b + b^3)$

(v) $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

(vi) $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

3 $(a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$

$$4 \quad a^3 + b^3 + c^3 - 3abc = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$(i) a^m \times a^n = a^{m+n} \quad (ii) a^m / a^n = a^{m-n}$$

$$(iii) (a^m)^n = a^{mn} \quad (iv) a^{-n} = 1/a^n$$

$$(v) (a/b)^n = a^n / b^n \quad (vi) \sqrt[n]{a} = a^{\frac{1}{n}}$$

WORKED OUT PROBLEMS

1. Simplify $36 \times 15 - 50 \times 784 \div 112$

$$\begin{aligned} \text{Answer : } 36 \times 15 - 50 \times 784 \div 112 \\ = 36 \times 15 - 50 \times 7 = 540 - 350 = 190 \end{aligned}$$

2. Simplify $900 \div [44 - \{(8-6) + (24-12)\}]$

$$\begin{aligned} \text{Answer} &= 900 \div [44 - \{(8-6) + (24-12)\}] \\ &= 900 \div [44 - \{2 + 12\}] \\ &= 900 \div [44 - 14] = 900 \div 30 = 30 \end{aligned}$$

3. Simplify $6 + 7 \div 7 + 4 \times 2 - 9 \div 9 \times 2 - 5$

$$\begin{aligned} \text{Answer : } 6 + 7 \div 7 + 4 \times 2 - 9 \div 9 \times 2 - 5 \\ = 6 + 1 + 8 - 1 \times 2 - 5 = 15 - 2 - 5 = 8 \end{aligned}$$

4.Simplify $11-(5-1)+\frac{1}{2}$ of $6 - 10 + 5 - 11$

$$\begin{aligned}\text{Answer : } 11-(5-1)+\frac{1}{2} \text{ of } 6 - 10 + 5 - 11 \\ = 11-(4)+3 - 10 + 5 - 11 = -6\end{aligned}$$

5.Simplify $x - [x - (y+x) - \{x - (\overline{x - y - x})\}] + 2y$

$$\begin{aligned}\text{Answer : } x - [x - (y+x) - \{x - (\overline{x - y - x})\}] + 2y \\ = x - [x - y - x - \{x - (x - y + x)\}] + 2y \\ = x - [-y - (x - x + y - x)] + 2y \\ = x - [-y + x - y] + 2y = 4y\end{aligned}$$

6.Simplify $\frac{658 \times 658 \times 658 - 328 \times 328 \times 328}{658 \times 658 + 658 \times 328 + 328 \times 328}$

$$\text{Answer: } \frac{(658)^3 - (328)^3}{658^2 + 658 \times 328 + 328^2} = 658 - 328 = 330$$

$$\text{Since } \frac{a^3 - b^3}{a^2 + ab + b^2} = a - b$$

7.Simplify $6-[6-\{6-6(6+6)\}]$

$$\begin{aligned}\text{Answer : } 6-[6-\{6-6(6+6)\}] \\ = 6-[6-\{6-6(12)\}] \\ = 6-[6-\{6-72\}] = 6-[6 + 66] = - 66.\end{aligned}$$

8 Simplify $81 \times 81 + 68 \times 68 - 2 \times 81 \times 68$

$$\begin{aligned}\text{Answer : } 81 \times 81 + 68 \times 68 - 2 \times 81 \times 68 \\ = 81^2 + 68^2 - 2 \times 81 \times 68 = (81-68)^2 = 13^2 = 169\end{aligned}$$

Since $(a - b)^2 = a^2 - 2ab + b^2$

9.Simplify $\frac{2}{5}$ of $45 \div 3 + [(9-4) \div 5]$

Answer : $\frac{2}{5} \times 45 \div 3 + [(5) \div 5]$
 $= 18 \div 3 + 1 = 6 + 1 = 7$

10.Simplify $5 - [6 - \{12 - (10 - \overline{8 - 4})\}]$

Answer : $5 - [6 - \{12 - (10 - \overline{8 - 4})\}]$
 $= 5 - [6 - \{12 - (10 - 4)\}] = 5 - [6 - \{12 - (6)\}]$
 $= 5 - [6 - 6] = 5 - 0 = 5$

EXERCISE

1. If $132 \times x - 225 = 171$ then find the value of 'x' [3]
2. Find the value of $6 - [4 - \{12 - (10 - \overline{8 - 4})\}]$ is
 _____ [8]
3. If $\frac{3}{8}$ of $\{3840 \div 8\} = a$ then find the value of 'a' [180]
4. If $a + b + c = 0$ then $a^3 + b^3 + c^3 =$ _____ [3abc]
5. Find the value of $[(4444 \div 44) - \{625 \div 25 + 1\}] \div 25$ _____ [3]
6. $(696 \div 4 - x) \times 5 = 20$ then find 'x' [170]

Web Resources for advanced Learners

1. <https://www.youtube.com/watch?v=70cAYYCJBuQ>
2. <https://www.twinkl.co.in/teaching-wiki/bodmas>
3. <https://www.youtube.com/watch?v=dAgfnK528RA>
4. <https://www.vedantu.com/maths/bodmas-rule>

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1.2. FRACTIONS

LEARNING OUT COMES:

After completion of this topic, students will be able to understand.

1. A fraction and types of fractions.
2. Simplification of fraction using BODMAS rule.

INTRODUCTION

Any number is of the form

$\frac{a}{b}$, where a,b are integers and $b \neq 0$ is called a fraction. Here 'a' is called numerator and 'b' is called denominator.

I.e. Fraction = $\frac{\text{Numerator}}{\text{Denominator}}$

Also we that rational number is known as a fraction.

$\frac{2}{5}, \frac{6}{10}, 2\frac{1}{5}$ etc. are examples of a fraction.

Types of a fractions

Type:1:: Proper fraction:

A fraction is said to be proper fraction if the numerator is less than the denominator.

Examples : $\frac{2}{5}, \frac{10}{17}$ etc.

Type:2 :: Improper fraction:

A fraction is said to be improper fraction if the numerator is greater than the denominator.

Examples : $\frac{5}{2}$, $\frac{17}{12}$ etc.

Type:3:: Mixed fraction:

Mixed fraction is a combination of whole number and a proper fraction.

An improper fraction can be written as a mixed fraction.

Examples : $2\frac{1}{5}$, $6\frac{2}{7}$ etc.

Note : Conversion of mixed fraction into improper fraction.

Take a mixed fraction namely $a\frac{b}{c}$. Improper fraction equal

to given mixed fraction is $\frac{ac+b}{c}$

Example: $2\frac{3}{5} = \frac{2 \times 5 + 3}{5} = \frac{13}{5}$

Mixed fraction	Improper fraction
----------------	-------------------

$$2\frac{1}{2}$$

$$\frac{5}{2}$$

$$6\frac{2}{5}$$

$$\frac{32}{5}$$

Type: 4: Compound fraction:

The fraction of a fraction is called a compound fraction

Example: $\frac{1}{2}$ of $\frac{4}{7} = \frac{1}{2} \times \frac{4}{7} = \frac{2}{7}$.

Type: 5: Complex fraction:

A fraction whose numerator or denominator or both are fractions, is called a complex fraction.

Examples : $\frac{2}{\frac{7}{4}}$, $\frac{7/5}{2}$, $\frac{2/5}{6/7}$ etc.

Type: 6: Continued fraction:

Continued fraction is a fraction that contains an additional fraction in the denominator or numerator.

Example : $2 + \frac{1}{1 + \frac{1}{2}}$

Note: A fraction whose denominator is multiples of 10 is called decimal fraction.

Examples : $\frac{3}{10} = 0.3$, $\frac{3}{100} = 0.03$

Algebraic Operations on fractions

Sum (Addition) of two fractions:

If the two fractions do not have the same denominator, reduce them to the common denominator, then sum of two fractions is fraction whose numerator is sum of the numerators and denominator is common denominator.

Example : $\frac{4}{3} + \frac{2}{5} = \frac{4}{3} \times \frac{5}{5} + \frac{2}{5} \times \frac{3}{3} = \frac{20}{15} + \frac{6}{15} = \frac{20+6}{15} = \frac{26}{15}$

Difference (Subtraction) of two fractions:

If the two fractions do not have the same denominator, reduce them to the common denominator, then difference of two fractions is fraction whose numerator is difference of the numerators and denominator is common denominator.

$$\text{Example: } \frac{4}{3} - \frac{2}{5} = \frac{4}{3} \times \frac{5}{5} - \frac{2}{5} \times \frac{3}{3} = \frac{20}{15} - \frac{6}{15} = \frac{20-6}{15} = \frac{14}{15}$$

Product (Multiplication) of two fractions:

The product of two fractions is a fraction whose numerator is the product of the numerators and denominator is the product of denominators.

$$\text{Example: } \frac{2}{5} \times \frac{6}{7} = \frac{12}{35}$$

Ratio (Division) of two fractions:

If a/b and c/d are two fractions, then their ratio is defined as

$$\frac{a/b}{c/d} = \frac{a}{b} \times \frac{d}{c} = \frac{ad}{bc}. \text{ Here } b \neq 0 \text{ and } d \neq 0.$$

$$\text{Example: } \frac{2/5}{3/8} = \frac{2}{5} \times \frac{8}{3} = \frac{16}{15}.$$

Note: If two mixed fractions are to be added (or subtracted) then first convert mixed fraction into improper fraction and proceed as above definition.

WORKED OUT PROBLEMS

1. Find the value of $3\frac{2}{3} + 2\frac{3}{4} + 1\frac{1}{2}$

$$\begin{aligned}\text{Answer : } 3\frac{2}{3} + 2\frac{3}{4} + 1\frac{1}{2} &= \frac{11}{3} + \frac{11}{4} + \frac{3}{2} \\ &= \frac{44}{12} + \frac{33}{12} + \frac{18}{12} = \frac{44+33+18}{12} = \frac{95}{12} = 7\frac{11}{12}.\end{aligned}$$

2. Find the value of $\frac{\frac{2}{3} \times \frac{3}{4}}{\frac{2}{4} \times \frac{8}{3}}$

$$\text{Answer : } \frac{\frac{2}{3} \times \frac{3}{4}}{\frac{2}{4} \times \frac{8}{3}} = \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} \times \frac{3}{2} = \frac{3}{4}$$

3. Find the value of $7\frac{1}{2} - [2\frac{1}{4} + \{1\frac{1}{4} - \frac{1}{2}(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6})\}]$

$$\begin{aligned}\text{Answer: } 7\frac{1}{2} - [2\frac{1}{4} + \{1\frac{1}{4} - \frac{1}{2}(1\frac{1}{2} - \frac{1}{3} - \frac{1}{6})\}] \\ &= \frac{15}{2} - [\frac{9}{2} + \{\frac{5}{4} - \frac{1}{2}(\frac{3}{2} - \frac{1}{3} - \frac{1}{6})\}] \\ &= \frac{15}{2} - [\frac{9}{2} + \{\frac{5}{4} - \frac{1}{2}(\frac{6}{6})\}] = \frac{15}{2} - [\frac{9}{2} + \{\frac{5}{4} - \frac{1}{2}\}] = \frac{15}{2} - [\frac{9}{2} + \{\frac{12}{4}\}] \\ &= \frac{15}{2} - \frac{6}{2} = \frac{9}{2} = 4\frac{1}{2}.\end{aligned}$$

4. Find the value of $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{100}\right)$

Answer : $\left(1 - \frac{1}{3}\right)\left(1 - \frac{1}{4}\right)\left(1 - \frac{1}{5}\right) \dots \left(1 - \frac{1}{100}\right)$

$$= \frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \dots \times \frac{99}{100} = \frac{2}{100} = \frac{1}{50}.$$

5. Find the value of $1 + \frac{\frac{1}{2}}{\frac{\frac{1}{3} + \frac{\frac{1}{4}}{\frac{1}{5} + \frac{1}{6}}}}$

$$\text{Answer: } 1 + \frac{\frac{1}{2}}{\frac{\frac{1}{3} + \frac{\frac{1}{4}}{\frac{1}{5} + \frac{1}{6}}}} = 1 + \frac{\frac{1}{2}}{\frac{\frac{1}{3} + \frac{1}{\frac{1}{3} + \frac{4}{11}}}} = 1 + \frac{\frac{1}{2}}{\frac{1}{\frac{1}{3} + \frac{30}{44}}} = 1 + \frac{\frac{1}{2}}{\frac{134}{132}} = 1 + \frac{1}{2} \times \frac{132}{134} = \frac{100}{67}$$

6. If $2 = x + \frac{1}{1 + \frac{1}{3 + \frac{1}{4}}}$ then find the value of 'x'.

$$\text{Answer : } x + \frac{1}{1 + \frac{1}{3 + \frac{1}{4}}} = 2 \Rightarrow x + \frac{1}{1 + \frac{1}{\frac{13}{4}}} = 2 \Rightarrow x + \frac{1}{1 + \frac{4}{13}} = 2$$

$$\Rightarrow x + \frac{1}{\frac{17}{13}} = 2 \Rightarrow x + \frac{13}{17} = 2 \Rightarrow x = 2 - \frac{13}{17} = \frac{21}{17}.$$

EXERCISE

1. Value of $2\frac{1}{5} + 1\frac{2}{5} - 3\frac{2}{5} =$ _____ [1/5]
2. Find the value of $\frac{1}{3}$ of $\frac{6}{7} + \frac{3}{2} \div 6$ [11/28]
3. Find the value of $6\frac{5}{12} \times 3\frac{3}{8} + 2\frac{3}{16} \times 3\frac{1}{2} -$ [$29\frac{5}{32}$]
a) $29\frac{5}{32}$ b) $29\frac{3}{32}$ c) $29\frac{1}{32}$ d) None
4. If $6\frac{2}{3} + 3\frac{3}{5} + 3\frac{5}{6} - x = 0$ then $x =$ _____ [$14\frac{1}{10}$]
5. If $2 = x + \frac{1}{1 + \frac{1}{3 + \frac{1}{4}}}$ then find the value of x . [$\frac{21}{17}$]
6. $\frac{2}{5}$ of $\frac{3}{7}$ of $\frac{5}{9}$ of $\frac{7}{24}$ of 504 = _____ [14]

Web Resources for advanced Learners

1. <https://www.youtube.com/watch?v=qmfXyR7Z6Lk>
2. <https://www.youtube.com/watch?v=GvLIEiqxS6s>
3. <https://www.youtube.com/watch?v=UacpcSkBhbM>
4. <https://www.cuemath.com/numbers/fractions/>

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1.3 DIVISIBILITY RULES

LEARNING OUT COMES:

After completion of this topic, students will be able to understand

1. Different divisibility rules
2. Simplify the problem based on divisibility rules.

INTRODUCTION

We have a certain rule that a number is divisible by 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 etc.

Now we require the following definitions to understand the divisibility rules.

Divisibility by 2:

A number is divisible by '2' if it's units place is even (i.e. multiple of 2)

Example 120, 134, 2456 etc.

Divisibility by 3:

A number is divisible by 3 if the sum of all digits is a multiple of 3.

Example: 369, 4356 etc.

Divisibility by 4:

A number is divisible by 4 if the last two digits are divisible by 4.

Example: 62512, 4224 etc.

Divisibility 5:

A number is divisible by 5 if it's last digit is 0 or 5

Example: 625, 1230 etc.

Divisibility by 6:

A number is divisible by 6 if it is divisible by both 2 and 3.

Example: 4356, 8586 etc.

Divisible by 7:

A number of three or more digits is divisible by 7 if the sum of the numbers formed by last two digits and twice the number formed by the remaining digits is divisible by '7'.

Example: 1792 is divisible by 7

Answer: $92 + 17 \times 2 = 92 + 34 = 126$ is divisible by 7

$\Rightarrow$ 1792 is divisible by 7.

Divisibility by 8:

A number is divisible by 8 if the number formed by last three digits is divisible by 8.

Example: 9216 is divisible by 8

Since 216 is exactly divisible by 8 ($216 = 8 \times 27$)

Divisibility by 9:

A number is divisible by 9 if the sum of its digits is divisible by 9.

Example: 243, 2232 etc.

Divisibility by 10:

A number is divisible by 10 if it has zero in its unit's place. (number ends with zero)

Example: 360, 3790 etc.

Divisibility by 11:

A number is divisible by 11 if the difference of the sum of the digits in even places and the sum of the digits in odd place is '0' or multiple of 11.

Example: 2838 is divisible by 11

Since the sum of digits in odd places $2 + 3 = 5$

The sum of digits in even places $8 + 8 = 16$

Difference $16 - 5 = 11$ is divisible by 11.

Divisibility by 12:

A number is divisible by 12 if it is divisible by both 3 and 4.

Example: 648, 144 etc.

Remainder theorem:

If ' r ' is a remainder when $x \times y \times z$ is divisible by ' n ' and r_1, r_2, r_3 are the remainder when x, y, z are divisible by ' n ' then r is equal to the remainder when $r_1 x r_2 x r_3$ is divisible by ' n '.

Example:

What is the remainder when $15 \times 25 \times 11$ is divisible by '6'?

Answer: r_1 = the remainder when 15 is divisible by 6 = 3

r_2 = The remainder when 25 is divisible by 6 = 1

r_3 = The remainder when 11 is divisible by 6 = 5

r = The remainder when $3 \times 1 \times 5 = 15$ is divisible by 6 is 3

WORKED OUT PROBLEMS:

1) Is 6244 is divisible by (i) by 2 , (ii) by 3

Answer : Given number is 6244

(i) The given number is ending with 4 (multiple of 2)
Then this number is divisible by 2

(ii) Sum of digits of given number is $6+2+4+4=16$ is
not divisible by 3

$\Rightarrow$ Given number is not divisible by 3

2) Is 36782 is divisible by 4.

Answer: The given number is 36782

The last two-digit number is 82 is not divisible by 4

$\Rightarrow$ Given number is not divisible by 4.

3) Which of the following numbers is divisible by 10,5,2 .

(i) 372

(ii) 3720

(iii) 2560

Answer:

(i) The first number in 372

This is divisible by 2 only not 5, 10

(ii), (iii) numbers are divisible by 2, 5, 10

Since the number ending with zero

4) Divide 0.0256 by 16

Answer: we have $\frac{256}{16} = 16$

$$\Rightarrow \frac{0.0256}{16} = 0.0016$$

5) Divide 35 by 0.005

Answer: we have $\frac{35}{5} = 7$

$$\Rightarrow \frac{35}{0.005} = \frac{35 \times 1000}{0.005 \times 1000} = \frac{3500}{5} = 700$$

6) Find the remainder when $1753 \times 87 \times 171$ is divisible by 17

Answer:

r_1 = The remainder when 1753 is divisible by 17 is 2

r_2 = The remainder when 87 is divisible by 17 is 2

r_3 = The remainder when 171 is divisible by 17 is 1

r = The remainder when $1753 \times 87 \times 171$ is divisible by 17

The remainder when $2 \times 2 \times 1 = 4$ is divisible by 17 = 4

EXERCISE

1. Which of the following number is not divisible by 11 [d]
a) 417 45 b) 21 73 16 c) 19756 d) 153980
2. Which of the following number is divisible by 2 [d]
a) 12345 b) 12453 c) 21435 d) 14352
3. How many numbers between 11 and 21 which are divisible by 5 [a]
a) 2 b) 3 c) 1 d) None
4. The number of numbers which are divisible by 6 which are lie between the numbers 237 and 250 are [b]
a) 1 b) 2 c) 3 d) None
5. The number 2160 not divisible by [d]
a) 2 b) 3 c) 5 d) 7
6. The number 1782 is divisible by [d]

a) 4

b) 5

c) 10

d) 11

Web Resources for advanced Learners

1. <https://brilliant.org/wiki/proof-of-divisibility-rules/>
2. <https://www.youtube.com/watch?v=xmaXdamlz1Y>
3. <https://www.youtube.com/watch?v=qGdug5Co00A>
4. https://www.youtube.com/watch?v=rIOgLe_plbs

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1.4. LEAST COMMON MULTIPLE (LCM)

Learning outcomes:

- 1.To enable students to compute LCM of given numbers.
- 2.To understand the concept of LCM and related results.
- 3.To apply the concept of LCM to practical situations when and where required.

Introduction:

In this section, a concept which is related to the multiples of given numbers will be discussed. We start with it's meaning, mathematical definition, results , relation between LCM and HCF followed by solved examples. This section ends with exercises.

Meaning of LCM:

‘LCM’ is the short form of ‘Least Common Multiple’.

Given two numbers a and b,

Find sets of multiples of a and b, i.e $\{a \times 1, a \times 2, \dots\}$,
 $\{b \times 1, b \times 2, \dots\}$

Sort out common multiples of a and b

Find the smallest(or)least among common multiples of a and b. This is designated as $LCM(a,b)$.

Note: For given two numbers LCM always exists and it is unique.

Example.1:

Find $LCM(8,12)$

Set of multiples of 8 is $\{8 \times 1, 8 \times 2, \dots\} = \{8, 16, 24, \dots\}$

Set of multiples of 12 is $\{12 \times 1, 12 \times 2, \dots\} = \{12, 24, 36, \dots\}$

Set of common multiples is $\{24, 48, 72, \dots\}$

Least among common multiples is 24

$$\therefore LCM(8,12) = 24.$$

Note: LCM concept can also be extended to more than two numbers. First find LCM of any two numbers and then find then LCM of the result and 3rd number. Continuing this process , LCM of given numbers can be found.

Example.2:

Find $LCM(8,12,18)$.

From Ex.1 , $LCM(8,12) = 24$

Now to find $LCM(24,18)$

Sets of multiples of 24 and 18 are

$\{24,48,72, \dots\}$, $\{18,36,54,72, \dots\}$

Set of common multiples is $\{72,144, \dots\}$

$$\therefore LCM(24,18) = 72$$

$$\therefore LCM(8,12,18) = 72.$$

Mathematical definition of LCM:

$LCM(a, b)$ is that number which is divisible by a and b and also divides any other common multiple of a and b .

Note: 1. $LCM(1,B)=B$, for any number B .

2. $LCM(A,A)=A$, for any number A

Methods to find LCM:

1.Prime factorization method:

Express each of the given numbers as the product of prime numbers. Then the product of prime numbers with greatest exponents gives the LCM of given numbers.

Example.1: Find $LCM(8,12)$

$$8 = 2^3, \quad 12 = 3 \times 2^2$$

$$\text{Then } LCM(8,12) = 3 \times 2^3 = 3 \times 8 = 24.$$

2. Common Division Method:

Arrange the given numbers in a row in any order .
Now divide by a prime number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat this process till no numbers have a common factor other than one . Then the product of the divisors and the remaining numbers gives the LCM of the given numbers .

Example.1 Find the LCM(8, 12)

2	8, 12
2	4, 6
	2, 3

$$\therefore LCM(8,12) = 3 \times 2 \times 2 \times 2 = 3 \times 8 = 24.$$

Example.2: Find LCM(8,12,18)

2	8, 12, 18
2	4, 6, 9
3	2, 3, 9
	2, 1, 3

$$\therefore LCM(8,12,18) = 3 \times 1 \times 2 \times 3 \times 2 \times 2 = 9 \times 8 = 72.$$

Some results:

1. The least number which when divided by x , y and z leaves the same remainder R in each case is given by

$$LCM(x, y, z) + R$$

Proof: Let the required number be 'm'. then

$m = xq_1 + R, m = yq_2 + R, m = zq_3 + R$, where q_1, q_2, q_3 are quotients when 'm' is divided by x, y, z respectively

$$\Rightarrow m - R = xq_1, m - R = yq_2, m - R = zq_3$$

$$\Rightarrow m - R \text{ is divisible by } x, y \text{ and } z$$

Since least of this property is required,

$$m - R = LCM(x, y, z)$$

$$\therefore m = LCM(x, y, z) + R$$

Hence the required number is $m = LCM(x, y, z) + R$

2. The least number which when divided by x , y and z leaves the remainders a , b and c respectively is given by

$$LCM(x, y, z) - p, \text{ where } p = x - a = y - b = z - c.$$

Proof: Let the required number be 'm'. then

$m = xq_1 + a, m = yq_2 + b, m = zq_3 + c$, where q_1, q_2, q_3 are quotients when 'm' is divided by x, y, z respectively

$$\Rightarrow m - a = xq_1, m - b = yq_2, m - c = zq_3$$

Since $p = x - a = y - b = z - c$

$$m - x + x - a = q_1 x$$

$$m - x + p = q_1 x$$

$$m + p = q'_1 x, \text{ where } q'_1 = q_1 + 1$$

$$\text{similarly } m + p = q'_2 y, m + p = q'_3 z,$$

$$\text{where } q'_2 = q_2 + 1 \text{ and } q'_3 = q_3 + 1$$

$$\Rightarrow m + p \text{ is a common multiple of } x, y, z$$

Since least of this property is required, $m + p = LCM(x, y, z)$

$$\therefore m = LCM(x, y, z) - p$$

Hence the required number is $m = LCM(x, y, z) - p$.

3. To find LCM of decimals ,first make them like decimals and find LCM as if they are whole numbers . Then put the decimal accordingly.

WORKED OUT PROBLEMS

1. Find LCM of 24,36 and 40.

Using division method 3 | 24, 36, 40

2	8, 12, 40
2	4, 6, 20
2	2, 3, 10
	1, 3, 5

$$\therefore LCM = 5 \times 3^2 \times 1 \times 2^3 = 5 \times 9 \times 8 = 360.$$

2. Compute LCM of $2^3 \times 3^2 \times 5 \times 11$, $2^4 \times 3^2 \times 5^2 \times 7$, $2^5 \times 3^3 \times 5^2 \times 7^2 \times 11$.

Since given numbers are in the form of product of prime numbers

$$LCM = 2^5 \times 3^3 \times 5^2 \times 7^2 \times 11 = 32 \times 27 \times 25 \times 49 \times 11 = 1,16,42,400.$$

3. Find LCM of 22, 54, 108, 135, 198.

Using division method	2	22, 54, 108, 135, 198
	3	11, 27, 54, 135, 99
	3	11, 9, 18, 45, 33
	3	11, 3, 6, 15, 11
	11	11, 1, 2, 5, 11
		1, 1, 2, 5,

$$\therefore LCM = 5 \times 2^2 \times 11 \times 3^3 = 5 \times 4 \times 11 \times 27 = 5940.$$

4. Find the least number which when divided by 20, 25, 35 and 40 leaves remainders 14, 19, 29 and 34 respectively.

Given numbers are 20, 25, 35, 40

Remainders are 14, 19, 29, 34

Note that $20 - 14 = 25 - 19 = 35 - 29 = 40 - 34 = 6 = p$

Then the required number = $LCM(20, 25, 35, 40) - p = LCM(20, 25, 35, 40) - 6$

Now to find $LCM(20, 25, 35, 40)$,

5	20, 25, 35, 40
4	4, 5, 7, 8
	1, 5, 7, 2

$$LCM(20, 25, 35, 40) = 2 \times 7 \times 5^2 \times 4 = 56 \times 25 = 1400$$

$\therefore$ Required number is $1400 - 6 = 1394$.

5. Find the number which when divided by 10 leaves remainder 9, when divided by 9 leaves remainder 8 and when divided by 8 leaves remainder 7.

Given numbers are 10, 9, 8

Remainders are 9, 8, 7

Note that $10 - 9 = 9 - 8 = 8 - 7 = 1 = p$

Then the required number = $LCM(10, 9, 8) - p = LCM(10, 9, 8) - 1$

Now to find $LCM(10, 9, 8)$, $10 = 5 \times 2$, $9 = 3^2$, $8 = 2^3$

$$LCM(10, 9, 8) = 2^3 \times 3^2 \times 5 = 8 \times 9 \times 5 = 360$$

The required number = $LCM(10, 9, 8) - 1 = 360 - 1 = 359$.

6. Six bells ring at intervals of 2, 3, 10, 15, 20, 22 seconds respectively. In 30 minutes duration, how many times all the bells ring together?

Given that six bells ring at an intervals of 2, 3, 10, 15, 20, 22 seconds respectively

To find the time taken by all the bells to ring simultaneously, we have to find $LCM(2, 3, 10, 15, 20, 22)$

2	2, 3, 10, 15, 20, 22
3	1, 3, 5, 15, 10, 11
5	1, 1, 5, 5, 10, 11
	1, 1, 1, 1, 2, 11

$$\therefore LCM(2, 3, 10, 15, 20, 22) = 11 \times 2 \times 5 \times 3 \times 2 = 660 \text{ SECONDS} = 11 \text{ MINUTES}$$

Which implies that six bells ring together at an interval of 11 minutes. In 30 minutes duration, six bells ring together for 2 times.

EXERCISES

- Find LCM of $\frac{6}{5}, \frac{5}{4}, \frac{4}{3}, \frac{3}{2}$.
- Find LCM of 2m 30cms, 3m 25cms.

3. Find the least number which when divided by 5, 7, 9 and 11 leaves remainders 2, 4, 6 and 8 respectively.
4. If two numbers are in the ratio 17: 8 and their HCF is 19 then find their LCM.
5. If three numbers which are coprime to each other are such that the product of first two is 551 and that of the last two is 1073. Then find the sum of the three numbers.
6. Find the least number that is exactly divisible by 1, 2, 3, 4, 5, 6, 7, 8, 9.
7. Which of the following fractions is the largest?

$$\frac{7}{8}, \frac{13}{16}, \frac{31}{40}, \frac{63}{80}$$

ANSWERS

1.60 2.149.5m 3.3462 4. 2584 5.85 6.2520 7. $\frac{7}{8}$

WEBRESOURCES:

[https://en.wikipedia.org/wiki/Least common multiple](https://en.wikipedia.org/wiki/Least_common_multiple)

<https://www.coursehero.com/study-guides/prealgebra/finding-the-least-common-multiple-of-two-numbers/>

<https://math.stackexchange.com/questions/44836/rational-numbers-lcm-and-hcf>

1.5 GREATEST COMMON DIVISOR

(GCD)

HIGHEST COMMON FACTOR (HCF)

Introduction:

In this section, a concept which is related to the factors of given numbers will be discussed. We start with its meaning, mathematical definition followed by methods to find HCF and results.

First, we start with meaning of HCF along with examples.

Meaning of HCF:

HCF is the short form of “Highest Common Factor”.

Given two numbers say a and b . Find factors (or) divisors of a and b separately. Sort out common factors (or) divisors of a and b . Find the highest among common factors of a and b .

This highest common one is designated as

$HCF(a,b)$. $HCF(a,b)$ is also known as $GCD(a,b)$

i.e Greatest Common Divisor of a and b .

Note: For given two numbers HCF always exists and it is unique.

Example.1: Find $HCF(8,12)$

Set of factors of 8 is $\{1,2,4,8\}$ and Set of factors of 12 is $\{1,2,3,4,6,12\}$

Set of common factors is $\{1,2,4\}$

Highest among common factors is 4

$$\therefore HCF(8,12) = 4$$

Note: HCF concept can also be extended to more than two numbers. First find HCF of any two numbers and then find then HCF of the result and 3rd number. Continuing this process ,HCF of given numbers can be found.

Example.2: Find $HCF(8,12,18)$

From Ex.1 , $HCF(8,12)=4$

So now to find $HCF(4,18)$,

Set of factors of 4 is $\{1,2,4\}$ and Set of factors of 18 is $\{1,2,3,6,9,18\}$

Set of common factors is $\{1,2, \}$

Highest among common factors is 2.

$$\therefore HCF(8,12,18) = 2.$$

Mathematical definition of HCF:

HCF(a,b) is that number which divides a and b and is divisible by any common factor of a and b.

Note: 1. $HCF(1,b)=1$ for any number b.

2. $HCF(a,b)=1$ if a,b are prime numbers.

Coprime numbers:

If $HCF(a, b) = 1$ then a,b are said to be Coprime numbers.

Note: Obviously any two prime numbers are Coprime numbers. But converse need not be true. For example, $HCF(4,9) = 1$ but 4,9 are not prime numbers.

Methods to find HCF

1.Prime factorization method:

Write each of given numbers as the product of prime numbers.

Then the product of common factors with least exponents gives the HCF of given numbers.

Note: If there are no common factors then HCF would be equal to 1.

Example: Find HCF(8,12)

$$8 = 2^3, 12 = 3 \times 4 = 3 \times 2^2$$

$$\therefore HCF(8,12) = 2^2 = 4.$$

2.Division method:

For two numbers a, b , suppose $a < b$. Then using division algorithm

$$b = aq_1 + r_1, 0 \leq r_1 < a$$

$$a = r_1q_2 + r_2, 0 \leq r_2 < r_1$$

Continue this process till the remainder is zero. Then the divisor in the last step is the required HCF.

Example:

1. Find HCF(8,12), $8 < 12$

$$12 = 8 \times 1 + 4$$

$$8 = 4 \times 2$$

$$\therefore HCF(8,12) = 4.$$

2. Find HCF(8,12,18)

From ex.1, $HCF(8,12)=4$

For 4,18, $4 < 18$

$$18 = 4 \times 4 + 2$$

$$4 = 2 \times 2$$

$$\therefore HCF(4,18) = 2$$

$$\therefore HCF(8,12,18) = 2.$$

RESULTS ABOUT HCF:

1. The greatest number that will divide x , y and z leaving remainders a , b and c respectively is given by $HCF(x - a, y - b, z - c)$.

Proof: Let the required number be 'd'

Using division algorithm

$x = dq_1 + a$, $y = dq_2 + b$, $z = dq_3 + c$, where q_1, q_2, q_3 are quotients when x, y, z are divided by 'd' respectively.

$$\Rightarrow x - a = dq_1, y - b = dq_2, z - c = dq_3$$

$$\Rightarrow d \text{ divides } x - a, y - b, z - c \text{ exactly}$$

Since greatest number of this property is required,

$$d = HCF(x - a, y - b, z - c)$$

Hence the required number 'd' is the HCF of

$$x - a, y - b, z - c.$$

2. The greatest number that will divide x , y and z leaving the same remainder in each case is given by

$HCF(x - y, y - z, z - x)$ where the differences are taken to be positive.

Proof: Let the required number be 'd'

Using division algorithm

$x = dq_1 + r$, $y = dq_2 + r$, $z = dq_3 + r$, where q_1, q_2, q_3 are quotients when x, y, z are divided by 'd' respectively. Here r is the remainder in each case.

$$\Rightarrow x - y = d(q_1 - q_2), y - z = d(q_2 - q_3), z - x = d(q_3 - q_1)$$

$$\Rightarrow d \text{ divides } x - y, y - z, z - x \text{ exactly}$$

Since greatest number of this property is required,

$$d = HCF(x - y, y - z, z - x)$$

Hence the required number 'd' is the HCF of $x - y, y - z, z - x$.

3. The greatest number that will divide x, y leaving the same remainder R in each case is given by $HCF(x - R, y - R)$.

Proof: Let the required number be 'd'

Using division algorithm

$x = dq_1 + R$, $y = dq_2 + R$, where q_1, q_2 are quotients when x, y are divided by 'd' respectively. Here R is the remainder in each case.

$$\Rightarrow x - R = dq_1, y - R = dq_2$$

$$\Rightarrow d \text{ divides } x - R, y - R$$

Since greatest number of this property is required,

$$d = HCF(x - R, y - R)$$

Hence the required number 'd' is the HCF of $x - R, y - R$.

4. To find the HCF of decimal numbers,

Given decimals are converted in to like decimals wherever necessary. Then find the HCF of the numbers without taking the decimal point in to consideration i.e treating them as whole numbers. The number of decimal places in the answer are equal to the number of decimal places in the given numbers. Accordingly put the decimal point in the answer.

Relationship between HCF and LCM of two numbers:

1.The product of HCF and LCM of two numbers a and b is equal to the product a.b

$$\text{i.e } HCF(a, b) \times LCM(a, b) = a \times b$$

2.HCF and LCM of fractions:

Given fractions are $\frac{a}{b}, \frac{c}{d}$ then

$$LCM\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{LCM \text{ of Numerators}}{HCF \text{ of Denominators}} = \frac{LCM(a,c)}{HCF(b,d)}$$

$$HCF\left(\frac{a}{b}, \frac{c}{d}\right) = \frac{HCF \text{ of Numerators}}{LCM \text{ of Denominators}} = \frac{HCF(a, c)}{LCM(b, d)}$$

Worked out problems

1. Find (i) $HCF(42,63)$ (ii) $HCF(108,288,360)$

(i) $42 < 63 \Rightarrow 63 = 42 \times 1 + 21$

$$42 = 21 \times 2$$

$$\therefore HCF(42,63) = 21$$

- (ii) First to find $HCF(108,288)$, $108 < 288$

$$288 = 108 \times 2 + 72$$

$$108 = 72 \times 1 + 36$$

$$72 = 36 \times 2$$

$$\therefore HCF(108,288) = 36$$

Now to find $HCF(36,360)$, $36 < 360$

$$360 = 36 \times 10$$

$$\therefore HCF(108,288,360) = 36.$$

2. Compute HCF of $2^3 \times 3^2 \times 5 \times 7^3$ and $2^3 \times 3^2 \times 5^2 \times 7^2$.

Note that numbers are given in the form of product of prime numbers.

$$\text{Hence } HCF = 2^2 \times 3^2 \times 5 \times 7^2 = 8820.$$

3. Find the largest number that divides 62,132,237 to leave the same remainder in each case.

Required number is $HCF(132 - 62, 237 - 132, 237 - 62)$
 $= HCF(70, 105, 175)$

First to find $HCF(70, 105)$, $70 < 105 \Rightarrow 105 = 70 \times 1 + 35$

$$70 = 35 \times 2 \Rightarrow HCF(70, 105) = 35$$

Now to find $HCF(35, 175)$, $35 < 175 \Rightarrow 175 = 35 \times 5$

$$\therefore HCF(70, 105, 175) = 35$$

Hence the Required number is 35.

4. What is the largest number that will divide 1657 and 2037 leaving the remainder 6 and 5 respectively?

Required number is $HCF(1657 - 6, 2037 - 5) =$
 $HCF(1651, 2032)$

$$1651 < 2032 \Rightarrow 2032 = 1651 \times 1 + 381$$

$$1651 = 381 \times 4 + 127$$

$$381 = 127 \times 3$$

$$HCF(1651, 2032) = 127$$

$\therefore$ Required number is 127.

5. Find the greatest number that divides 261, 933, 1381 leaving the remainder 5 in each case.

Required number is $HCF(933 - 261, 1381 - 933, 1381 - 261)$

$$= HCF(672, 448, 1120)$$

First to find $HCF(672, 448)$, $448 < 672 \Rightarrow 672 = 448 \times 1 + 224$

$$448 = 224 \times 2$$

$$\therefore HCF(672, 448) = 224.$$

Now to find $HCF(224, 1120)$, $224 < 1120 \Rightarrow 1120 = 224 \times 4 + 224$, $224 = 224 \times 1$

$$\therefore HCF(224, 1120) = 224.$$

$$\therefore HCF(672, 448, 1120) = 224.$$

Therefore the required number is 224.

6. If the sum of two numbers is 204 and their HCF is 17 then find all possible pairs.

Let the two numbers be a and b . Given that $HCF(a, b) = 17$

$$\text{then } a = 17x, \quad b = 17y \text{ where } HCF(x, y) = 1$$

$$\text{Given } a + b = 204 \Rightarrow 17x + 17y = 204$$

$$x + y = \frac{204}{17} = 12$$

$$\Rightarrow x + y = 12$$

This is to find x, y such that $x + y = 12$ and $HCF(x, y) = 1$

Possible pairs are $x = 1, y = 11$ then $a = 17, b = 187$

$$x = 5, y = 7 \text{ then } a = 85, b = 119$$

Required pairs are (17,187) and (85,119).

7. The product of two numbers is 6300 and their HCF is 15. Then how many pairs of such numbers are possible ?

Let the two numbers be a and b . Given that $HCF(a, b) = 15$

$$\text{then } a = 15x, \quad b = 15y \text{ where } HCF(x, y) = 1$$

$$\text{Given } a \times b = 6300 \Rightarrow 15x \times 15y = 6300$$

$$xy = \frac{6300}{225} = 28$$

$$xy = 28$$

Now to find x, y such that $xy = 28$ and $HCF(x, y) = 1$

$$\Rightarrow x=4 \text{ and } y=7 \text{ then } a = 15 \times 4 = 60 \text{ and } b = 15 \times 7 = 105$$

Therefore Possible pair is (60,105).

8. Find HCF of 3.6, 0.24, 1.2

Take the given numbers as 3.60, 0.24, 1.20

Then given numbers can be considered as 360, 24, 120

Now to find $HCF(360, 24)$, $24 < 360 \Rightarrow 360 = 24 \times 15$

$$HCF(360, 24) = 24$$

To find $HCF(24,120)$, $24 < 120$, $\Rightarrow 120 = 24 \times 5$

$$HCF(24,120) = 24$$

Since after decimal point, there are two digits

$$\text{Therefore } HCF(3.6, 0.24, 1.2) = 0.24.$$

EXERCISES

- Find HCF of
 - 1785, 1995
 - 1672, 374
 - 30, 42, 135
- Find the greatest number which will divide 2112 and 2792 leaving the remainder 4 in each case.
- What is the greatest number that will divide 964, 1238, 1400 leaving remainders 41, 31, 51 respectively?
- Compute the HCF of $2^4 \times 3^5 \times 6^3 \times 7^2$, $2^5 \times 3^2 \times 6 \times 7^5$ and $2^3 \times 3^4 \times 6^2 \times 7 \times 9^2$.
- How many pairs exist such that their HCF is 6 and their sum is 42? Also find the pairs.
- What is the largest number that will divide 50, 90, 110 leaving remainders 8, 6, 5 respectively?

7. Find HCF of $x^5 \times y^2 \times z, v^2 \times w \times x^4, u \times v \times w^2$ where x, y, z, u, v, w are prime numbers.
8. Find the product of two numbers if their whose LCM and HCF are 289 and 17 respectively.
9. Three different containers contain different quantities of mixtures of water and sugar whose measurements are 225, 300 and 330 litres. what is the biggest measure to measure all different quantities exactly?

ANSWERS

1. (i) 105 (ii) 2 (iii) 3
2. 68 3.71 4. $2^5 \times 3^3 \times 7$
- 5 3 pairs and (6, 36), (12,30), (18,24)
- 6 21
- 7 1
- 8 4913
- 9 15

1.6 : SERIES OF NUMBERS

Learning Outcomes :

After completion of the topic student can able to

1. Identify the different series of numbers and patterns,
2. Understand the logic and solve the problems.
3. Apply and enhances problem solving skill,

4. To use logical thinking and improve analytical ability,
5. Solve quantitative aptitude questions for different competitive exams.

An Arithmetical value used to represent quantity is known as a **Number**.

It is used to count, measure and label. We have different types of numbers Systems such as

Set of Natural Numbers denoted by $N = \{1, 2, 3, \dots\}$

Set of Whole numbers denoted by $W = \{0, 1, 2, 3, \dots\}$

Set of Integers, denoted by $Z = \{\dots, -2, -1, 0, 1, 2, \dots\}$ and Real Numbers.

A Number consists of digits, where 253, 67, and 1055 are the numbers consisting

three digits, two digits, and four digits respectively. Numbers are many types, they are

Even numbers: Numbers which can be divided by 2 or all multiples of 2.

example: -4, -2, 2, 4, 6, 8, 10, 12, 14 are even numbers.

236 is an even number where as 351 is not an even number since it is not divisible by 2 completely.

Odd numbers: Numbers which are not divisible by 2.

example: -5, -3, -1, 1, 3, 5, 7, 9, 11 are odd numbers.

555, 107, 337 are some odd numbers.

Prime numbers : The Numbers which are divisible by 1 and itself are called “ Prime numbers”. In other words the numbers which have no factors other than 1 and itself. Otherwise they are called Composite numbers.

example: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31 and etc are Prime numbers.

1 is neither Prime number Nor composite number.

The number 2 is an only even prime.

Series: Arrangement of numbers in a definite order is said to be a series.

Series are many types, they are

1. Addition series
2. Subtraction series
3. Multiplication series
4. Division Series
5. Square series
6. Fibonacci series and
7. Alternating series.

Addition series : Each term of the series is obtained by adding a fixed number to next term.

Example

1) 6, 9, 12, 15, 18, 21, ----.

- a) 26 b) 28 c) 33 d) 24 [d]

Each term of the above series is obtained by adding 3 to its before term.

2) 5, 10, 15, 25, 40, 65, 105, -----.

- a) 111 b) 170 c) 165 d) 185 [b]

Third term is addition of previous two numbers.

Subtraction series: Each term of the series is obtained by Subtracting a fixed number from the before term.

Example

1) 195, 190, 180, 175, 165, 160, -----.

- a) 155 b) 158 c) 150 d) 140 [c]

Each term of the above series is obtained by subtracting 5,10 alternatively from before terms.

2) 220, 216, 208, 196, 180, -----.

a) 160 b)165 c) 168 d) 150 [a]

Each term of the above series is obtained by subtracting the multiples of 4 from before terms.

Multiplication series: Each term of the series is obtained by multiplying before number with a fixed number to get the next term.

Example

1) 5, 10, 20, 30, 40, 50, 60, -----.

a)80 b) 70 c) 90 d)140 [b]

2) 220, 216, 208, 196, 180, -----.

a) 160 b)165 c) 168 d) 150 [a]

Division series: In a given series every term is obtained on dividing previous term with a fixed value called division series.

Example

1) 800, 400, 200, -----, 50.

a)80 b) 70 c) 100 d)140 [c]

2) 81, 27, 9, -----, 1.

a) 12 b)16 c) 11 d) 3 [d]

Square series: In this series the terms are squares of a number.

Example

- 1) 2, 4, 3, 9, 4, 16, 5, ----, ----, 36.
a) 25, 6 b) 6, 26 c) 25, 6 d) 14, 10 [c]
- 2) 36, 49, 64, 81, -----.
a) 160 b) 100 c) 168 d) 150 [b]
- 3) 1, 8, 27, 64, ----
a) 125 b) 100 c) 168 d) 150 [a]
- 4) 144, 169, 196, 225, ----
a) 160 b) 256 c) 168 d) 150 [b]

Fibonacci series: In a series the third term is the sum of its previous two terms.

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, --, --, -- is the Fibonacci series.

Alternating series: In this series multiple operators are applied.

1, 6, 2, 7, 3, 8, 4, 9, 5, -10, --. is an alternating series.

EXERCISE

Complete the series with a suitable answer.

1. 2, 3, 8, 27, 112, ----

- a) 505 b) 555 c) 523 d) 565

2. 1, 2, 6, 15, 31, 56, 92, -----

- a) 140 b) 145 c) 144 d) 141

3. 4, 64, 6, 36, 5, 125, 9, 81, 8, 512, 11, -----.

- a) 120 b) 124 c) 121 d) 112

4. 1, 2, 4, 7, 11, 16, 22, 29, ----.

- a) 33 b) 35 c) 31 d) 37

5. 0, 3, 8, 15, 24, 35, -----.

- a) 41 b) 46 c) 48 d) 44

Which number will come in the place of “ ? ”

6.

1	3	6
7	6	8
6	3	?

- a) 8 b) 14 c) 9 d) 2

7.

4	11	6
7	?	5
2	1	3

- a) 8 b) 14 c) 12 d) 13
8. 94, 76, 67, 85, 904, 508, ?.
- a) 1084 b) 88 c) 1008 d) 1801
9. 7, 14, 21, 28, 35, 42, ?.
- a) 48 b) 45 c) 49 d) 42
10. 6, 12, 24, 48, 96, ?.
- a) 1084 b) 194 c) 193 d) 192

Find the wrong number of the series.

11. 5, 7, 21, 23, 69, 70, 213.
a) 23 b) 7 c) 71 d) 69
12. 5, 69, 150, 250, 371, 514, 684.
a) 150 b) 514 c) 69 d) 684
13. 17, 36, 53, 83, 92.
a) 36 b) 83 c) 53 d) 92
14. 30, 42, 56, 76, 90, 110.
a) 76 b) 56 c) 110 d) 42
15. 1, 1, 2, 3, 5, 9, 13, 21.
a) 9 b) 5 c) 21 d) 3

ANSWERS :

1. d 2. d 3. C 4. D 5. c
6. d 7. C 8. A 9. c 10. d
11. c 12. B 13. B 14. A 15. a

WEB LINKS :

1. <https://oeis.org/>
2. <https://www.studyandscore.com>
3. <https://www.indiabix.com>
4. <https://scoop.eduncle.com>
5. <https://www.vedantu.com>
6. <http://www.theonlinetestcentre.com>



1.7. CODING-DECODING

LEARNING OUTCOMES:

To enable the students to -

- Know and understand the concepts of Coding & decoding

- Ability to visualize, gather information, articulate, analyse, solve complex problems from this Chapter
- Understand the concepts of coding and decoding
- Analyze the data from the information collected, and come up with a solution to a problem of Coding and decoding

Introduction:

Coding is a process used to encrypt a word, a number in a particular code or pattern based on some set of rules.

Decoding is a process to decrypt the pattern into its original form from the given codes. Position of Letters in English Alphabet

Types of Coding - Decoding

Type 1: Letter/Number Coding

Order of the English Alphabet

Forwa rd	Lette rs	Backwa rd	Forwa rd	Lette rs	Backwa rd
---------------------	---------------------	----------------------	---------------------	---------------------	----------------------

Order Position		Order Position	Order Position		Order Position
1	A	26	14	N	13
2	B	25	15	O	12
3	C	24	16	P	11
4	D	23	17	Q	10
5	E	22	18	R	9
6	F	21	19	S	8
7	G	20	20	T	7
8	H	19	21	U	6
9	I	18	22	V	5
10	J	17	23	W	4
11	K	16	24	X	3
12	L	15	25	Y	2
13	M	14	26	Z	1

Letter Coding In this type of questions, alphabets of a word are replaced by some other alphabets according to specific rule to form code.

Number Coding In this type of questions, a word is replaced by certain numbers according to some specific rule.

Example 1: In a certain code language, 'GIVE' is written as 'VIEG' and 'OVER' is written as "EVRO". How will 'DISK' be written in that same code?

- (a) SIDK
- (b) KISD
- (c) KDSI
- (d) SIKD
- (e) None of these

Solution: (d) ,

C	F	I	L	O	R	U	X
↓	↓	↓	↓	↓	↓	↓	↓
3	6	9	12	15	18	21	24
(Sounds like a medicine name)							
E	J	O	T	Y			
↓	↓	↓	↓	↓			
5	10	15	20	25			
(Sounds like a girl's name Joti)							

Example 2: If in a certain code 'INTELLIGENCE' is written as 'ETNIGILLECNE', then how can 'MATHEMATICAL' be written in the same code?

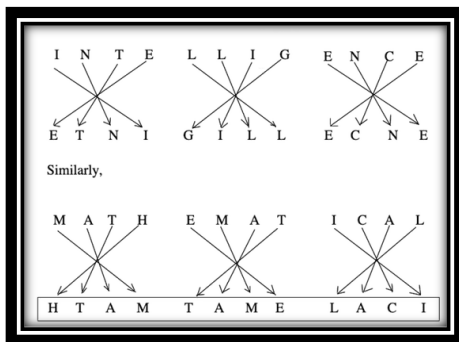
(a) AMHTMETACILA

(b) TAMMEHTALAC

(c) HTAMTAMELACI

(d) LACITAMEHTAM

Solution: (c) A



Example 3: If GOODNESS is coded as HNPCODTR, then how GREATNESS can be written in that code?

(a) HQZSMFRT

(b) HQFZUFRTM

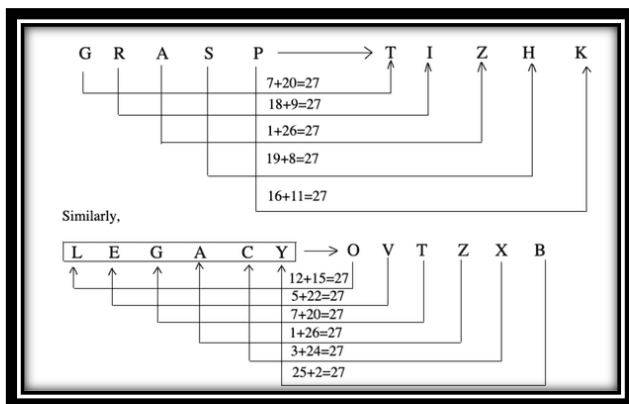
(c) HQFZUODTR

(d) HQFZUMFRT

Solution: (d) Clearly, the letters in the word GOODNESS are moved one step forward and one step backward, alternately to obtain the letters of the code.

Example 4: If GRASP is code as TIZHK, what will be code as OVTZXB?

- (a) LEGATE (b) LEAGUE
(c) LEGACY (d) LEDGER



Solution: (c)

Example 5: If 'WORK' is coded as '4-12-9-16', then how will you code 'WOMAN'?

- (a) 4-12-14-26-13 (b) 4-26-14-13-12
(c) 23-12-26-14-13 (d) 23-15-13-1-14

Solution: (a) ,

		Then,	
W	→ 4	W	→ 4
O	→ 12	O	→ 12
R	→ 9	M	→ 14
K	→ 16	A	→ 26
		N	→ 13

Here, each letter is coded by the numerical value obtained by subtracting its 'position value' in English alphabetical order, from 27, e.g. letters W, O, M, A, N are at 23rd, 15th, 13th, 1st and 14th position in alphabetical order. So, their codes are

(27 - 23), (27 - 15), (27 - 13), (27 - 1), (27 - 14), i.e. 4, 12, 14, 26, 13 respectively i.e. their backward order position.

Example 6: If CUP = 40, then KITE = ?

(a) 10 (b) 20

(c) 30 (d) 45

Solution: (d) Using forward letter positions,

	3	21	16	
As,	C	U	P	$\Rightarrow 3 + 21 + 16 = 40$
Similarly,				
	11	9	20	5
	K	I	T	E
	$\Rightarrow 11 + 9 + 20 + 5 = \boxed{45}$			

Example 7: If BAG = 71, then VICE = ?

(a) 69 (b) 70

(c) 75 (d) 90

Solution: (a) Using backward letter positions,

	25	26	20	
As,	B	A	G	$\Rightarrow 25 + 26 + 20 = 71$
Similarly,				
	V	I	C	E
	↓	↓	↓	↓
	5	18	24	22
	$= 5 + 18 + 24 + 22 = \boxed{69}$			

Type #2: Direct Coding

In direct coding, the code letters/numbers/ symbols occur in the same sequence as the corresponding letters occur in the words. This is basically a direct substitution method.

Example 8: If DEVICE is coded as BAREXA and ORNATE is coded as ZVMESA, what VIDEO be coded as?

- (a) RIBAZ (b) REBAZ
(c) RBEAZ (d) ZABER

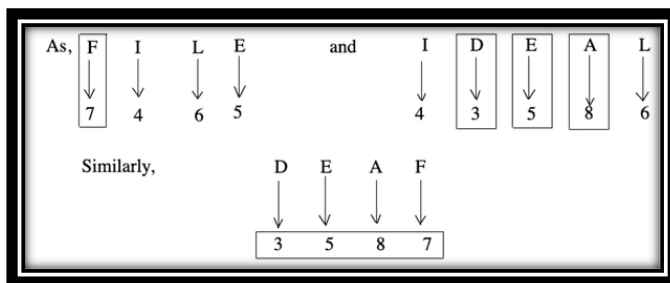
Solution: (d) Using direct letter coding method,

As,	$D \longrightarrow B$	and	$O \longrightarrow Z$	Similarly,	$V \longrightarrow$	$\begin{matrix} R \\ E \\ B \\ A \\ Z \end{matrix}$
	$E \longrightarrow A$		$R \longrightarrow V$		$I \longrightarrow$	$\begin{matrix} E \\ B \\ A \\ Z \end{matrix}$
	$V \longrightarrow R$		$N \longrightarrow M$		$D \longrightarrow$	$\begin{matrix} E \\ B \\ A \\ Z \end{matrix}$
	$I \longrightarrow E$		$A \longrightarrow E$		$E \longrightarrow$	$\begin{matrix} E \\ B \\ A \\ Z \end{matrix}$
	$C \longrightarrow X$		$T \longrightarrow S$		$O \longrightarrow$	$\begin{matrix} E \\ B \\ A \\ Z \end{matrix}$
	$E \longrightarrow A$		$E \longrightarrow A$			$\begin{matrix} E \\ B \\ A \\ Z \end{matrix}$

Example 9: In a certain code language, FILE is written as 7465 and IDEAL is written as 43586. How will DEAF be written in that code language?

- (a) 3478 (b) 3588
(c) 3587 (d) 4578

Solution: (d) Using direct number coding method,



Example 9: In a certain code, 'GONE' is written as 5 % 2 # and 'MEDAL' is written as 4 # 3 \$ @, then how will 'GOLD' be written in that code?

- (a) 5 @ % 3
(b) 5 % @ 3
(c) 5 # @ 3
(d) 5 % # 3

Solution: (b) Using direct coding method,

As,	G ↓ 5	O ↓ %	N ↓ 2	E ↓ #	and	M ↓ 4	E ↓ #	D ↓ 3	A ↓ \$	L ↓ @				
Similarly, <table style="display: inline-table; border: 1px solid black; padding: 5px; margin-left: 20px;"> <tr> <td style="padding: 5px;">G ↓ 5</td> <td style="padding: 5px;">O ↓ %</td> <td style="padding: 5px;">L ↓ @</td> <td style="padding: 5px;">D ↓ 3</td> </tr> </table>											G ↓ 5	O ↓ %	L ↓ @	D ↓ 3
G ↓ 5	O ↓ %	L ↓ @	D ↓ 3											
Therefore, GOLD => 5 % @ 3														

Type 3: Conditional Coding

In this type, letters/numbers are given along with their code. Candidates are required to find the codes for a particular letter group/word/number according to certain conditions given in the questions.

Directions (Examples 11 - 13) Study the following letters and their corresponding digits code followed by certain conditions of coding and then answer the questions given below them by finding out which of the digits combinations given in (a), (b), (c) and (d) is the coded form of the letters-groups given in each question and mark your answer accordingly.

Letters	P	N	A	J	I	R	E	B	U	K
Digits/Codes	5	3	9	1	4	6	2	7	0	8

Conditions

(I) If both the first and the last letters in the group are vowels, both should be coded as \$.

(II) If both the first and the last letters in the group are consonants, both should be coded as #.

Example 11: KUNAJB

(a) 803917

(b) \$0391\$

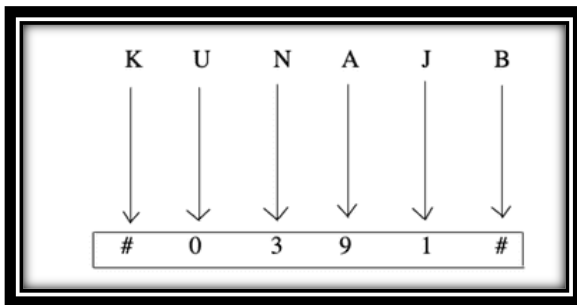
(c) #0391#

(d) #0391\$

(e) None of these

Solution: (c) We know that, in English alphabets A, E, I, O, U letters are vowels and remaining letters are consonants.

From condition (II),



Example 12: RBUKAE

(a) #70892

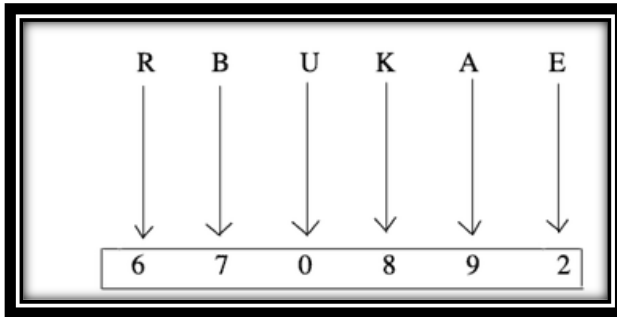
(b) 670892

(c) 670982

(d) 607892

(d) None of these

Solution: (b) We know that, in English alphabets A, E, I, O, U letters are vowels and remaining letters are consonants.



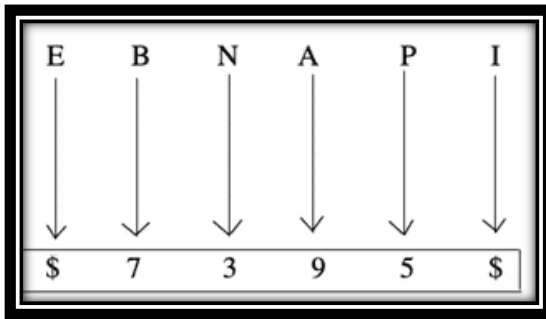
Note This question does not follow any condition.

Example 13: EBNAPI

- (a) 273954
- (b) \$7395\$
- (c) #7395#
- (d) \$7395#
- (e) None of these

Solution: (b) We know that, in English alphabets A, E, I, O, U letters are vowels and remaining letters are consonants.

From condition (I),



Type #4: Fictitious Language Coding

Here, coded form of two or more sentences is given and you are required to find the code of a particular word or message.

To analyses such codes, any two messages bearing a common word/number are picked up. The common code will represent that particular word/number.

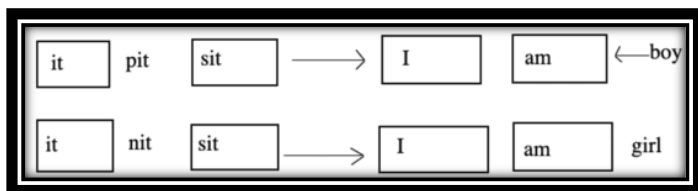
Proceeding similarly by picking up all possible combinations of two, the entire message can be decoded and the codes for every individual word/number can be found.

In some cases, there is no common word/number in the message given. In those cases, the words/numbers are coded separately. You have to identify the correct pattern of coding and find the code for the word or message given in the question.

Example 14: In a certain code language, 'it pit sit' means 'I am boy', 'it nit sit' means 'I am girl', which of the following means 'girl'?

- (a) it
- (b) pit
- (c) sit
- (d) nit
- (e) None of these

Solution: (d) according to the question, we have



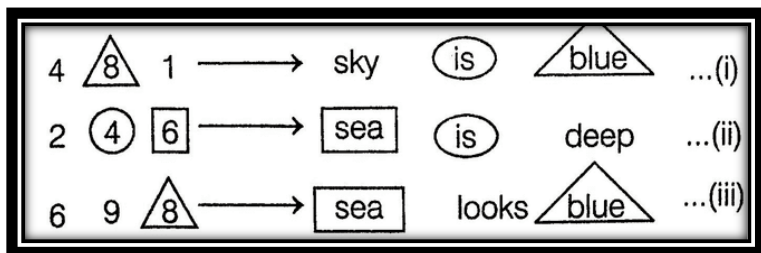
Here, 'it' and 'sit' are common in both the codes and 'I' and 'am' are common in both messages.

Hence, code for girl will be 'nit'.

Example 15: In a certain code language, 481 means 'sky is blue', 246 means 'sea is deep' and 698 means 'sea looks blue'. What number is the code for 'blue'?

- (a) 1 (b) 6 (c) 8 (d) 9

Solution: (c) according to the question, we have



From Eqs. (i) and (ii), we see that the code for blue is '8'.

Directions (Examples 16 - 17): Study the following information carefully and answer the questions given below.

In a certain code language

'given time simple plan' is written as

'@E4 & N4 % N5 # E6'

'tried solution plant great' is written as

'#N8 @ D5 % T5 & T5'

'sick point good turn' is written as

'#k4 % D4 @ N4 & T5'

'garden sister phone team' is written as

'&E5 # R6 % N6 @ M4'

Example 16: Which of the following is the code for
'translate'?

(a) @E84

(b) @E9

(c) #E8

(d) #T8

(e) #T9

Solution: (b) Here, symbols are coded as per the first letter
of the word.

As for 'g' symbol is %.

similarly, 'p' ---> &, 't' ---->@ and 's' ----> #

The number in the code represents the number of letter in the word and the letter in the code represents the last letter of the word.

For Example: In the word "given" g = %, last letter = N and number of letters = 5

Therefore, Given = % N5

Code for translate -----> @E9

Example 17: In the given code language, what does the code '%D4' represent?

- (a) Point
- (b) Turn
- (c) Sick
- (d) Good
- (e) None of these

Solution: (d) Here, symbols are coded as per the first letter of the word.

As for 'g' symbol is %.

similarly, 'p' ---> &, 't' ---->@ and 's' ----> #

The number in the code represents the number of letter in the word and the letter in the code represents the last letter of the word.

e.g. In the word "given" g = %, last letter = N and number of letters = 5

Therefore, Given = % N5

% ----> g

So, from the options % D4 ----> Good

Type # 5: Substitution Coding

In this type of coding, some particular words are coded with certain substituted word and on the basis of substituted word, the code of given word is derived.

Example 18: If 'white' is called 'blue', 'blue' is called 'red', 'red' is called 'yellow', 'yellow' is called 'green', 'green' is called 'black', 'black' is called 'violet' and 'violet' is called 'orange', then what would be the colour of human blood?

Solution: (c) We know that, the colour of human blood is 'red' but here 'red' is called 'yellow'. So, the colour of human blood will be 'yellow'.

Example 19: If 'parrot' is known as 'peacock', 'peacock' is known as 'swallow', 'swallow' is known as 'pigeon' and 'pigeon' is known as 'sparrow', then what would be the name of Indian National Bird?

- (a) Parrot
- (b) Swallow
- (c) Peacock
- (d) Pigeon

Solution: (b) We know that, peacock is the Indian National Bird but here peacock is known as swallow.

So, the answer is Swallow.

Example 20: On another planet the local terminology for 'earth', 'water', 'light', 'air' and 'sky' are sky, light, 'air', water and 'earth' respectively. If someone is thirsty there what would he drink?

- (a) Light (b) Air (c) Sky (d) Water

Solution: (a) One drinks water when he is thirsty.

since, 'water' drink 'light' when he is thirsty there. So, the answer is 'light'.

Sample Problems –

Question 1: If EARTH is written as FCUXM in a certain code. How is MOON written in that code?

Solution :

E	A	R	T	H	M	O	O	N
+1	+2	+3	+4	+5	+1	+2	+3	+4
F	C	U	X	M	N	Q	R	R

Question 2: If DELHI is written as EDMGJ in a certain code. How is NEPAL written in that code?

Solution :

D	E	L	H	I	N	E	P	A	L
+1	+1	+1	+1	+1	+1	+1	+1	+1	+1
E	D	M	G	J	O	D	Q	Z	M

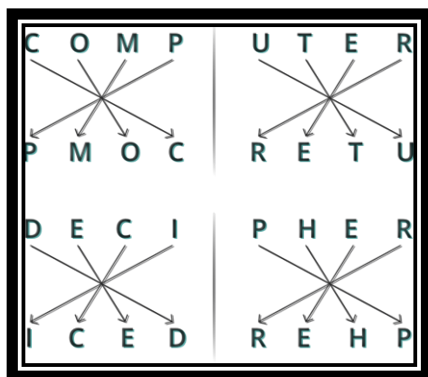
Question 3: If SYMBOL is written as NZTMPC is a certain code. How is NUMBER written in that code?

S	Y	M	B	O	L	N	U	M	B	E	R
+1					+1	+1					+1
N	Z	T	M	P	C	N	V	O	S	F	C

Solution :

Question 4: In a certain code, COMPUTER is written as PMOCRETU, how is DECIPHER written in that code?

Solution : According to question



Question 5: In a certain code, NEWYORK is written as 111, how is NEWJERSEY written in that code?

Solution : In NEWYORK

$N = 14, E = 5, W = 23, Y = 25, O = 15, R = 18, K = 11$

Total = $14 + 5 + 23 + 25 + 15 + 18 + 11 = 111$

In NEWJERSEY

Total = $14 + 5 + 23 + 10 + 5 + 18 + 19 + 5 + 25 = 124$

Question 6: In a certain code, HARYANA is written as 8197151, how is DELHI written in that code?

Solution : We used the number of alphabets here.

$H = 8$

$A = 1$

$R = 18 = 1 + 8 = 9$

$Y = 25 = 2 + 5 = 7$ $N = 1 + 4 + 5$

For DELHI

D = 4

E = 5

L = 12 = 1+2 = 3

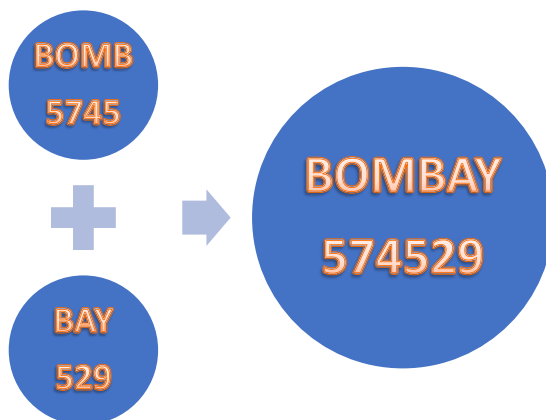
H = 8

I = 9

Hence, DELHI is written as 45389.

Question 7: In a certain code BOMB is written as 5745 and BAY is written as 529, how is BOMBAY written in that code?

Solution : Use the numbers to relate the words



EXERCISE

Q1: If in a certain language, MADRAS is coded as NBESBT, how is BOMBAY coded in that code?

Answer: CPNCBZ

Q2: In a certain code, TRIPPLE is written as SQHOOKD. How is DISPOSE written in that code?

Answer: CHRONRD

Q3: If in a code language. COULD is written as BNTKC and MARGIN is written as LZQFHM, how will MOULDING be written in that code?

Answer: LNTKCHMF

Q4: In a certain code, MONKEY is written as XDJMNL. How is TIGER written in that code?

Answer: QDFHS

Q5: If FRAGRANCE is written as SBHSBODFG, how can IMPOSING be written?

Answer: NQPTJOHJ

Q6: In a code, CORNER is written as GSVRIV. How can CENTRAL be written in that code?

Answer: GIRXVEP

Q7: If JOSEPH is coded as FKOALD, then GEORGE will be coded as:

(a) CAKNCA (b) CALNCA (c) CBKNCA (d) HAKNCA

Answer: CAKNCA

Q 8: In a certain language, if Violet is called as Green, Green is called as Red, Red is called as

Brown, Brown is called as Orange, Orange is called as Yellow, Yellow is called as Blue

and Blue is called as Indigo, then what is the colour of human blood in that language?

Answer: Brown

Q 9: If 'P' means 'division', 'T' means 'addition', 'M' means 'subtraction', and 'D' means

'multiplication', then what will be the value of the following expression?

$$12 \text{ M } 12 \text{ D } 28 \text{ P } 7 \text{ T } 15 = ?$$

Answer: -21

Q 10: The number in the question below is to be codified in the following code.

Digit: 5 3 7 1 4 9 6 2 8

Letter: C J O X N Q T Z F

Find the code for 163542

Answer: XTJCNZ

Scan me to get full text:

1.8 - Blood Relationship

Learning Outcomes:

To enable the students to -

- Know and understand the concepts of Blood relationship
- Ability to visualize, gather information, articulate, analyze, solve complex problems from this Chapter
- Understand the concepts of Blood relationship
- Analyze the data from the information collected, and come up with a solution to a problems of Blood relationship

Introduction:

Blood relation forms an important part of the reasoning section in most aptitude exams. Blood relation shows the **different relations among the members of a family**. Based on the information given, you are required to find the relation between particular members of the family.

Any relation in the world which either by birth or by marriage is called a Blood Relation. Ex: Any relation by birth will

be **mother, father, son, daughter**, etc. and any relation by marriage will be father-in-law, mother-in-law, etc.

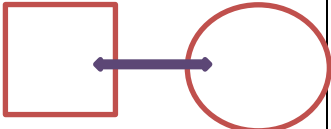
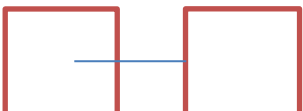
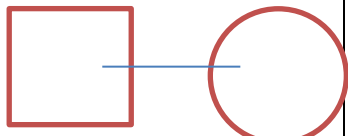
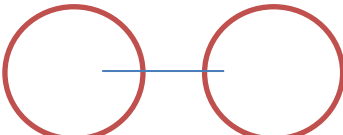
The word “**Souse**” which can be used for “**Husband**” or “**Wife**”

Relationships from “**Mother’s Side**” are prefixed by “**Maternal**” while relationships “**Father’s Side**” are prefixed by “**Paternal**”

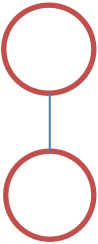
To develop blood relations, some standard symbols are used to indicate the relationship among the family members

However, there is no fixed rule to show the relations by symbolic representation.

S.No	Symbolic Representation	Relation/Meaning
1.		Male
2.		Female

3.		Husband -Wife
4.		Wife – Husband
5.		Brother – Brother
6.		Brother - Sister
7.		Sister - Sister

8.		Father – Son
9.		Father – Daughter
10.		Mother - Son

11.		Mother - Daughter

Some of the Common Relationships given as follows:

1. Relations of Paternal Side:

Father's father → Grandfather

Father's mother → Grandmother

Father's brother → Uncle

Father's sister → Aunt

Children of uncle → Cousin

Wife of uncle → Aunt

Children of aunt → Cousin

Husband of aunt → Uncle

2. Relations of Maternal Side:

Mother's father → Maternal grandfather

Mother's mother → Maternal grandmother

Mother's brother Maternal uncle

Mother's sister → Aunt

Children of maternal uncle → Cousin

Wife of maternal uncle → Maternal aunt

3. Relations from one generation to next:

Generation - I →

**Grand Father, Grand Mother,
Maternal Grand Father and
Maternal Grand Mother**

Generation – II →

**Mother, Father, Uncle,
Aunt, Maternal Uncle,
Maternal Aunt**

Generation – III →

**Self, Sister, Sister-in-Law, Brother,
Brother, Brother-in-Law**

Generation – III →

Son, daughter, nephew, niece

WORKED OUT PROBLEMS:

1. Pointing towards a photograph of a boy Suresh said, "He is the son of the only son of my mother." How is Suresh related to that boy?

Answer: Father

Explanation:

The boy in the photograph is the only son of the son of Suresh's mother i.e., the son of Suresh. Hence, Suresh is the father of boy.

2. If $A + B$ means A is the mother of B; $A - B$ means A is the brother B; $A \div B$ means A is the father of B and $A \times B$ means A is the sister of B, which of the following shows that P is the maternal uncle of Q?

a) $Q - N + M \times P$

b) $P + S \times N - Q$

c) $P - M + N \times Q$

d) $Q - S \div P$

Answer : (d) $Q - S \div P$

Explanation:

$P - M \rightarrow P$ is the brother of M

$M + N \rightarrow M$ is the mother of N

$N \times Q \rightarrow N$ is the sister of Q

Therefore, P is the maternal uncle of Q .

3. If A is the brother of B ; B is the sister of C ; and C is the father of D , how D is related to A ?

Answer: cannot be determined

Explanation:

If D is Male, the answer is Nephew.

If D is Female, the answer is Niece.

As the sex of D is not known, hence, the relation between D and A cannot be determined.

Note: Niece - A daughter of one's brother or sister, or of one's brother-in-law or sister-in-law. **Nephew** - A son of one's brother or sister, or of one's brother-in-law or sister-in-law.

4. If $A + B$ means A is the brother of B; $A - B$ means A is the sister of B and $A \times B$ means A is the father of B. Which of the following means that C is the son of M?

a) $M - N \times C + F$

b) $F - C + N \times M$

c) $N + M - F \times C$

d) $M \times N - C + F$

Answer : Option D

Explanation:

$M \times N \rightarrow$ M is the father of N

$N - C \rightarrow$ N is the sister of C

and $C + F \rightarrow$ C is the brother of F.

Hence, M is the father of C or C is the son of M.

5. Introducing a boy, a girl said, "He is the son of the daughter of the father of my uncle." How is the boy related to the girl?

Answer: Brother

Explanation:

The father of the boy's uncle $\rightarrow$ the grandfather of the boy
and daughter of the grandfather $\rightarrow$ sister of father.

c) Either (1) or (2)

d) (1) and (2) both

Answer : Option B

Explanation:

A and B are children of D.

From (1), C is the brother B and son of E.

Since, the sex of D and E are not known. Hence (1) is not sufficient to answer the question.

From (2). F is the mother of B. Hence, F is also the mother of A. Hence D is the father of A.

Thus, (2) is sufficient to answer the question.

8. Pointing towards a man, a woman said, "His mother is the only daughter of my mother." How is the woman related to the man?

Answer: Mother

Explanation:

Only daughter of my mother → myself.

Hence, the woman is the mother of the man.

9. If $P + Q$ means 'P is the brother of Q'; $P \times Q$ means 'P is the mother of Q'; and $P \div Q$ means 'P is the sister of Q', then which of the following would mean 'R is the uncle of S'?

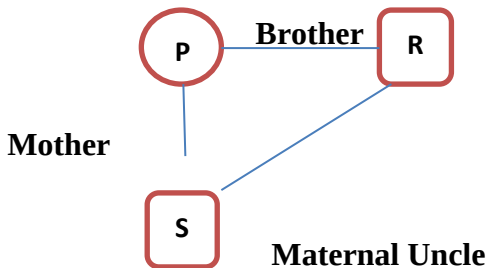
- a) $R \times P + S$ b) $R + P \times S$ c) $R \times S + S$
 d) $R + P + S$

Answer : Option A

Explanation:

$P \times S$ means, P is the mother of S, $R + P$ means, R is the brother of P.

This can be represented as



10. Introducing Sonia, Aamir says, "She is the wife of only nephew of only brother of my mother." How Sonia is related to Aamir?

Answer : Wife

Explanation

Brother of mother means maternal uncle. Hence only nephew of Aamir's maternal uncle means Aamir himself. Therefore Sonia is the wife of Aamir.

Exercise

How is U related P in the given expression?

$$U \# T \times S \div R \times Q + P$$

- (A) Sister (B) Husband (C) Wife (D) Mother-in-law

Answer: (C)

2. A is the brother of B. B is the brother of C. D is the father of A. Based on these three statements, which of the following statements cannot be definitely true?

- (A) A is the brother of C (B) C is the brother of A
(C) B is the brother of A (D) B is the son of D

Answer: (B)

3. If $A + B$ means A is the brother of B; $A - B$ means A is the sister of B and $A \times B$ means A is the father of B. Which of the following means that C is the son of M?

- (A) $M - N \times C + F$ (B) $F - C + N \times M$
(C) $N + M - F \times C$ (D) $M \times N - C + F$

Answer: (D)

4. If 'A×B' means that A is the sister of B, 'A÷B' means that A is the daughter of B, 'A–B' means that A is the son of B, then how is P related to S in relationship $P - Q \times R \div S$?

(A) Brother (B) Son (C) Grandson (D) None of these

Answer: (C)

5. If $A + B$ means A is the mother of B; $A \times B$ means A is the father of B; $A \$ B$ means A is the brother of B and $A @ B$ means A is the sister of B then which of the following means P is the son of Q?

(A) $Q + R @ P @ N$ (B) $Q + R * P @ N$
(C) $Q \times R \$ P @ N$ (D) $Q \times R \$ P \$ N$

Answer: (D)

6. A's Son B is married to C whose sister D is married to E, who is the brother of B. How is C related to E?

(A) Sister-in-law (B) Sister (C) Daughter (D) Cousin

Answer: (A)

7. P's father Q is B's paternal uncle and A's husband M is P's paternal uncle. How is A related to B ?

- (A) Mother
- (B) Cousin
- (C) Aunt
- (D) Data inadequate
- (E) None of these

Answer: (A)

8. Introducing a man, a woman says, "His wife is the only daughter of my father". How is the man related to the woman?

- (A) Brother
- (B) Father-in-law
- (C) Maternal uncle
- (D) Husband

Answer: D

9. Pointing to a girl Sandeep said, "She is the daughter of the only sister of my father." How is sandeep related to the girl?

- (A) Uncle
- (B) Cousin
- (C) Father
- (D) Grandfather

Answer: (B)

10. A visits his brother P who stays with their father Q, mother S, and grandfather R, P has two children, B, and C.

B's husband is X, and C's wife is Y. C's son T welcomes A on his visit. How is B related to Q?

(A) Grand-daughter (B) Grandson (C) Son (D) Daughter

Answer: (A)

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1.9 - Clocks

LEARNING OUTCOMES:

To enable the students to -

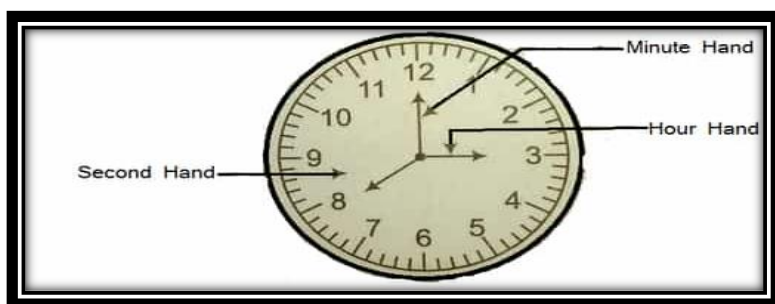
- Know and understand the concepts of Clocks
- Ability to visualize, gather information, articulate, analyze, solve complex problems from this Chapter
- Understand the concepts of Clocks
- Analyze the data from the information collected, and come up with a solution to a problems of Clocks

Introduction:

A clock's dial is like a complete circle that can be divided into 12 equal parts, with each division representing different hours of the day. Each hour space is further subdivided into 5 spaces – this represents duration of 1 minute.

There are typically 2 hands that rotate in a clock. Though in totality there are three hands, but the seconds hand is not that significant from the point of view of clocks aptitude questions. In an hour, the hour hand covers one of the 12 equal parts of the clock i.e. it covers $(1/12)$ th of the circumference

of the circle in an hour's time. On the other hand, the minute hand covers exactly 1 circumference of the clock in one hour. Further, we can conclude that the time in which the minute hand covers one full circle, the hour hand covers only $(1/12)$ th of the circumference.



The clock represents two things. i.e., minutes an hour. A minute is a unit of time equal to $1/60$ th of an hour or 60 s i.e, **1 min = 60 s.**

Minute Spaces:

The face or dial of watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.

Hour Hand and Minute Hand:

A clock has two hands, the smaller one is called the **hour hand** or **short hand** while the larger one is called **minute hand** or **long hand**.

- In 60 minutes, the minute hand gains 55 minutes on the hour on the hour hand.
- In every hour, both the hands coincide once.
- The hands are in the same straight line when they are coincident or opposite to each other.
- When the two hands are at right angles, they are 15 minute spaces apart.
- When the hands are in opposite directions, they are 30 minute spaces apart.
- Angle traced by hour hand in 12 hrs = 360°
- Angle traced by minute hand in 60 min. = 360° .
- If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes **too fast**.
- On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes **too slow**

Important Points for Clock problems:

- a) As minute hand covers one full circle i.e. 360 degrees in one hour, that means it travels $360/60 = 6$ degrees/min.
- b) An hour hand covers one part of the 12 major parts of the circle which means it covers $360/12 = 30$ degrees in one hour i.e. it travels $30/60 = 1/2$ degree per min.
- c) The complete circle has a total of 360 degrees and in terms of minute spaces, it has been divided into 60 minutes spaces, which means each minute space represents $360/60 = 6$ degrees.
- d) The complete circle has been divided into 12 equal bigger units also, which we call as hours. This further implies that every hour space covers a total of $360/12 = 30$ degrees.
- e) The hour hand and minute hand meet once every hour. But in a 12 hour period, they meet 11 times.
- f) There is one angle of 180 degrees in every hour i.e. both the hands are in the straight line in the opposite direction, but in a twelve hour period it happens 11 times.

- g) There are 2 right angles every hour, but in a 12 hour period there are 22 such angles.
- h) In 60 minutes, the minute hand gains 55 minutes on the hour hand
- i) In every hour, both the hands coincide once.
- j) The hands are in the same straight line when they are coincident or opposite to each other.
- k) When the two hands are at right angles, they are 15 minute spaces apart
- l) When the hand's are in opposite directions, they are 30 minute spaces apart

WORKED OUT PROBLEMS:

Type 1: To know time when angle is given

Example 1: At what time between 4 and 5, will the hands of a clock coincide?

Solution: At 4 O'clock, the hour hand has covered $(4 \times 30^\circ) = 120^\circ$.

To catch up with the hour hand, the minute hand has to cover a relative distance of 120° , at a relative speed of $5\frac{1}{2}^\circ$ per minute.

Thus, time required = $\frac{120 \times 2}{11} = \frac{240}{11}$ = or $21\frac{9}{11}$ minutes.

Example 2: At what time between 10 and 11 will the minute and hour hand be at right angles?

Solution: At 10 O'clock, the hour hand has covered $(10 \times 30^\circ) = 300^\circ$.

Note: There will be two right angles (clockwise and anti-clockwise)

Considering that hour hand is at 10, to make a 90-degree angle with the hour hand, the minute hand has to be at 1 or 7.

For the first right angle, minute hand has to cover a relative distance of $(1 \times 30) = 30^\circ$.

For the 2nd right angle, minute hand has to cover a relative distance of $(7 \times 30) = 210^\circ$.

We know that the relative speed between the two hands is of

$5\frac{1}{2}$ per minute.

Hence, time required for the 1st right angle = $\frac{30 \times 2}{11} = \frac{60}{11}$ or

$5 \frac{5}{11}$ minutes

Time required for the 2nd right angle = $\frac{210 \times 2}{11} = \frac{420}{11}$ or

$38 \frac{2}{11}$ minutes.

Type 2: To know angle when time is given

Example 1: The angle between the minute hand and the hour hand of a clock when the time is 4:20 is:

Solution: At 4:00, hour hand was at 120 degrees.

Using the concept of relative distance, the minute hand will

cover = $\frac{20 \times 11}{2} = 110$ degrees

The angle between the hour hand and minute hand is = $120 - 110 = 10$ degrees.

Example 2: The angle between the minute hand and the hour hand of a clock when the time is 3:30 is:

Solution: At 4:00, hour hand was at 90 degrees.

Using the concept of relative distance, the minute hand will

$$\text{cover} = \frac{30 \times 11}{2} = 165 \text{ degrees}$$

The angle between the hour hand and minute hand is = $165 - 90 = 75$ degrees.

To learn the tricks to solve the third type of questions asked from this topic, read our article on Clocks-Gaining/Losing of Time.

Type-3

1. How many times are the hands of a clock at right angle in a day?

Answer: In 24 hours, they are at right angles 44 times

2. How many times in a day, are the hands of a clock in straight line but opposite in direction?

Answer: In a day, the hands point in the opposite directions 22 times

3. The angle between the minute hand and the hour hand of a clock when the time is 4 : 20

Answer: 10°

4. At what time between 9 and 10 o'clock will the hands of a watch be together?

Answer: The hands are together at $49\frac{1}{11}$ min past 9

Type 4

1. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours will be the true time when the clock indicates 1 p.m. on the following day?

Answer: 48 min. past 12.

Explanation: Time from 8 a.m. on a day to 1 p.m. on the following day = 29 hours.

24 hours 10 min. of this clock = 24 hours of the correct clock.

$\frac{145}{6}$ hrs of this clock = 24 hours of the correct clock.

29 hours of this clock = $24 \times \frac{6}{145} \times 29$ hrs of the correct clock
= 28 hrs 48 min of the correct clock.

Therefore, the correct time is 28 hrs 48 min. after 8 a.m. This is 48 min. past 12.

2. At what time between 2 and 3 o'clock will the hands of a clock be together?

Answer: $(10 + 10/11)$ min past 2

Explanation:

At 2 o'clock, the hour hand is at 2 and the minute hand is at 12, i.e. they are 10 min spaces apart.

To be together, the minute hand must gain 10 minutes over the hour hand.

Now, 55 minutes are gained by it in 60 min.

10 minutes will be gained in $\frac{60}{55} \times 10$ min. = $10\frac{10}{11}$ min.

The hands will coincide at $10\frac{10}{11}$ min. past 2.

3. A clock is set right at 5 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?

Answer: 11 pm

Explanation: Time from 5 am. on a day to 10 pm. on 4th day
= 89 hours.

Now 23 hrs 44 min. of this clock = 24 hours of correct clock.

$\frac{356}{15}$ hrs of this clock = 24 hours of correct clock

89 hrs of this clock = $(24 \times \frac{356}{15} \times 89)$ hrs of correct clock.

= 90 hrs of correct clock.

So, the correct time is 11 p.m.

4. At what time between 4 and 5 o'clock will the hands of a watch point in opposite directions?

Answer: $(54 + \frac{6}{11})$ past 4

Explanation: 4 o'clock, the hands of the watch are 20 min. spaces apart.

To be in opposite directions, they must be 30 min. spaces apart.

Minute hand will have to gain 50 min. spaces.

55 min. spaces are gained in 60 min

50 min. spaces are gained in $\frac{60}{55} \times 50$ min. or $54\frac{6}{11}$

Required time = $54\frac{6}{11}$ min. past 4.

5. At what time between 5 and 6 o' clock are the hands of a 3 minutes apart?

Answer: 24min

Explanation: In this type of problems the formulae is

$$(5 \times x + \text{or} - t) \frac{12}{11}$$

Here x is replaced by the first interval of given time. Here x is 5. t is spaces apart

$$\text{Case 1 : } (5 \times x + t) \times \frac{12}{11}$$

$$\Rightarrow (5 \times 5 + 3) \times \frac{12}{11}$$

$$\Rightarrow 28 \times \frac{12}{11} = \frac{336}{11} = 31 \frac{5}{11} \text{ min}$$

Therefore the hands will be 3 min apart at $31 \frac{5}{11}$ min past 5.

Case 2 : $(5 \times x - t) \times \frac{12}{11}$

$$\Rightarrow (5 \times 5 - 3) \times \frac{12}{11}$$

$$\Rightarrow 22 \times \frac{12}{11} = 24 \text{ min}$$

Therefore, the hands will be 3 min apart at 24 min past 5

EXERCISE

1. The angle between the minute hour and the hour hand of a clock when the time is 8.30, is
(A) 60° (B) 70° (C) 75° (D) 80°

Answer: C

2. The angle between the minute hour and the hour hand of a clock when the time is 4.20, is
(A) 0° (B) 5° (C) 10° (D) 20°

Answer: C

3. A clock is set right at 8 a.m. The clock gains 10 minutes in 24 hours will be the true time when the clock indicates 1 p.m. on the following day?

- (A) 48 min past 12 (B) 46 min past 12 (C) 45 min past 12
(D) 47 min past 12

Answer: A

4. At what time between 2 and 3 o'clock will the hands of a clock be together?

- (A) $(9 + \frac{10}{11})$ min past 2 (B) $(10 + \frac{10}{11})$ min past 2
(C) $(11 + \frac{10}{11})$ min past 2 (D) $(12 + \frac{10}{11})$ min past 2

Answer: B

5. A clock is set right at 5 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?

- (A) 11pm (B) 12pm (C) 1pm (D) 2pm

Answer: A

6. At what time between 4 and 5 o'clock will the hands of a watch point in opposite directions?

- (A) 54 past 4 (B) $(53 + \frac{7}{11})$ past 4
(C) $(54 + \frac{8}{11})$ past 4 (D) $(54 + \frac{6}{11})$ past 4

Answer: D

7. At what time between 5 and 6 o' clock are the hands of 3 minutes apart?

- (A) 24min (B) 12min (C) 13min (D) 14min

Answer: A

8. A watch which gains uniformly ,is 5 min, slow at 8 o'clock in the morning on Sunday and it is 5 min 48 sec. fast at 8 p.m on following Sunday. when was it correct?

- (A) 7pm on Wednesday (B) 20 min past 7pm on Wednesday
(C) 15min past 7pm on Wednesday (D) 8pm on Wednesday

Answer: B

9. Find the angle between the hour hand and the minute hand of a clock when the time is 3.25

- (A) 47.5 degrees (B) 57.5 degrees (C) 45.5 degrees
(D) 55.5 degrees

Answer: A

10. In 16 minutes, the minute hand gains over the hour hand by – (A) 86° (B) 88° (C) 90° (D) 98°

Answer: B

1.10 CALENDARS

LEARNING OBJECTIVES:

After reading this topic the student will be able to

- Understand the concepts of ordinary year, Leap year, and odd days.
- Find the number of odd days in given months, years and centuries.
- Find the day of the week for a given date.

Introduction:

A calendar is a system of organizing the days in the form of weeks, months, and years. Ancient calendars remain in regional use for religious or social purposes whereas now in worldwide we are using Gregorian calendars for secular purposes.

Definitions:

Ordinary year: The year which has 365 days is called an ordinary year.

Leap year: The year which has 366 days (Having 29 days in the month of February) is called a leap year.

What are leap years:

- Any year other than centuries which are divisible by 4 (Examples: 1820, 1996, 2020, etc.) but in the case of centuries it should be divisible by 400 (Examples: 400,800,1200,1600,2000, etc. are leap years)
- The years 200,300,500,700, etc. are not leap years.

Odd days:

The remainder obtained by dividing the number of days by 7 is called odd days.

For example, if there are 365 days in a year if we divide 365 by 7 then we get the quotient as 52 and the remainder as 1. That means an ordinary year has 52 weeks and 1 extra day. That extra day is called an odd day.

∴ The number of odd days in an ordinary year is 1

Similarly, the number of odd days for the leap year is 2

Let us check the number of odd days for the different months. For example, in the month of January, we have 31 days. That means the month has 4 complete weeks and 3 extra days. So, the number of odd days for January is 3.

Table 1:

Sl.No	Month	Days	Odd days
1	January	31	3
2	February	28(29)	0(1)
3	March	31	3
4	April	30	2
5	May	31	3
6	June	30	2
7	July	31	3
8	August	31	3
9	September	30	2
10	October	31	3
11	November	30	2
12	December	31	3

Codes for the days:

Generally starting of the week is Monday so the code for Monday is 1 and the codes for other days are as follows.

Table 2:

Day	Sun	Mon	Tue	Wed	Thu	Fri	Sat
Code	0	1	2	3	4	5	6

Odd days for centuries:

A century is a period of 100 years. Now we will discuss the odd days for centuries.

In 100 years there will be 76 ordinary years and 24 leap years. For each ordinary year, there will be 1 odd day whereas for leap year 2 odd days.

Therefore the total odd days are $76(1)+24(2)=76+48=124$; divide 124 by 7 then the quotient is 17 and the remainder is 5
 $\therefore$ The no. of odd days in 100 years=5;

200 years= $5 \times 2=10$; (divide 10 by 7 the remainder is 3) $\therefore$ The no. of odd days in 200 years=3;

300 years= $5 \times 3 = 15$; (divide 15 by 7 the remainder is 1) $\therefore$ The no. of odd days in 300 years=1;

Table 3:

Sl.No.	Years/Centuries	Odd days	Last day of the century
1	100	5	Friday
2	200	3	Wednesday
3	300	1	Monday
4	400,800,1200,1600,2000,2400 etc..	0	Sunday
5	500(100+400)	5+0=5	Friday
6	600(200+400)	3+0=3	Wednesday
7	700(300+400)	1+0=1	Monday
8	800(400+400)	0+0=0	Sunday

NOTE: The last day of the century cannot be Tuesday, Thursday and Saturday.

WORKED OUT PROBLEMS:

1. Today is Friday, after 152 days what will be the day?

Solution: Divide 152 by 7 then the remainder is 5
[$152 = (21 \times 7) + 5$]

5 days beyond Friday is Wednesday

(Saturday, Sunday, Monday, Tuesday, Wednesday)

$\therefore$ The answer is Wednesday

2. Find the number of odd days in 1887 years.

Solution: $1887 = 1600 + 200 + 87$ years

$= 0 + 3 + 87 + 21$ (leap years)

$= 111$ (divide 111 by 7 then the remainder is 6)

$\therefore$ The number of odd days = 6.

3. Find the day of 15th October 1931 which is the birth date of Sri A.P.J. Abdul kalam.

Solution: $1931 = 1600 + 300 + 30$ years + January (31 days) + Feb (28) + March (31) + April (30) + May (31) + June (30) + July (31) + August (31) + Sep (30) + October (15 days)

$= 0 + 1 + 30 + 7 + 3 + 0 + 3 + 2 + 3 + 2 + 3 + 3 + 2 + 1 = 60$

(when 60 is divided by 7 then the remainder is 4)

Odd days = 4

Decode for 4 is Thursday

∴ The day on 15th October 1931 is Thursday.

Years/days	No. of odd days
1600 years	0
300 years	1
30years	30+7(as there are 7 leap years in 30 years)
January (31 days)	3
February(28 days)(as this is not leap year)	0
March(31 days)	3
April(30 days)	2
May(31 days)	3
June(30 days)	2
July(31 days)	3
August(31 days)	3
September(30 days)	2
October(15 days)	1(divide 15 by 7 the remainder is 1)

Total odd days	60
-----------------------	----

$$=0+1+30+7+3+0+3+2+3+2+3+3+2+1 =60$$

(when 60 is divided by 7 then the remainder is 4)

Odd days =4

Decode for 4 is Thursday

∴ The day on 15th October 1931 is Thursday.

4. What will be the day on 26th July 2024?

Solution: The number of odd days in 2021= The odd days in (2000years+23years+ the days from January 1st to 26th July 2024)

Year/Days	No. of odd days
2000 years	0
23 years	$23+5=28$ (5 leaps years in 23 years)
January (31 days)	3

February(29 days)(as this is a leap year)	1
March(31 days)	3
April(30 days)	2
May(31 days)	3
June(30 days)	2
July(26 days)	5(divide 26 by 7 the remainder is 5)
Total odd days	47

The number of odd days in 47=(divide 47 by 7; we get the remainder 5)=5

Decode for 5 is: Friday

5. August 15th, 2022 was Monday, what will be the Day on August 15th, 2026?

Solution: The number of odd days from August 15th, 2022 to August 15th, 2023 =1

The number of odd days from August 15th, 2023 to August 15th, 2024 = 2 (as 2024 is a leap year)

The number of odd days from August 15th, 2024 to August 15th, 2025 = 1

The number of odd days from August 15th, 2025 to August 15th, 2026 = 1

The total number of odd days = $1 + 2 + 1 + 1 = 5$

5 days beyond Monday is Saturday

∴ The day on August 15th 2026 = Saturday

6. If January 1, 2000, is Saturday then what was the day on Jan 1, 2003?

Solution: The number of odd days from Jan 1, 2000, to Jan 1, 2001, = 2 (since 2000 is a leap year)

The number of odd days from Jan 1, 2001, to Jan 1, 2002, = 1

The number of odd days from Jan 1, 2002, to Jan 1, 2003, = 1

The total number of odd days = $2 + 1 + 1 = 4$

4 days beyond Saturday is Wednesday

∴ The day on Jan 1, 2003 was Wednesday.

7. On what dates in August 1947 did Friday fall?

Solution: First find the day on 1st August 1947.

Years/days	No. of odd days
1600 years	0
300 years	1
46 years	46+11(as there are 11 leap years in 46 years)
January (31 days)	3
February(28 days)(as this is not a leap year)	0
March(31 days)	3
April(30 days)	2
May(31 days)	3
June(30 days)	2
July(31 days)	3
August(1 day)	1
Total odd days	75

The number of odd days in 75=(divide 75 by 7; we get the remainder 5)=5

Decode for 5 is: Friday

∴ August 1st, 1947 was Friday.

Thus Friday fell on the 1st, 8th, 15th, 22nd, and 29th of August 1947.

8. If 15th August 2022 was Monday, what will be the day on 26th January 2023?

Solution:

Month/Days	No. of days
August-2022(from Aug 16 ^h to Aug 31 st)	16
September-2022	30
October-2022	31
November-2022	30
December-2022	31
January-2023(1 st Jan to 26 th Jan)	26
Total no. of days	164

Divide 164 by 7 the remainder is 3.

3 days beyond Monday is Thursday.

∴ The day on **26th January 2023 is Thursday**

9. The calendar for the year 2021 will be the same as the year?

Solution:

Year	2021	2022	2023	2024	2025	2026	2027	2028	2029
No. of odd days	1	1	1	2(since it is a leap year)	1	1	1	2(since it is a leap year)	1

Add the odd days until the sum is multiples of 7.

In the above table, the sum up to 2026 is 7.

Therefore the Calendar for 2021 will be the same as the year 2027(The next year to 2026)

Note 1. The ordinary year starts and ends on the same day.

Example: In 2022(ordinary year) Jan 1st and Dec 31st both are on Saturday

2. The leap year ends one day beyond the starting day.

Example: In 2020(leap year) Jan1 is on Wednesday whereas Dec 31st is on Thursday.

10. If December 31, 2013, was Tuesday, then what was the day on Jan 1, 2013?

Solution: Given the year 2013 is an ordinary year.

We know that the ordinary year starts and ends on the same day.

∴ The day on Jan 1, 2013, was Tuesday.

11. If the Day on Jan 1, 1820, was Saturday, what was the day on Dec 31, 1820?

Solution: Given the year 1820 is a leap year.

We know that the leap year ends one day beyond the starting day.

One day beyond Saturday is Sunday.

∴ The day on Dec 31, 1820, was Sunday.

12. If the day on Dec31,2096 will be Monday, what will be the day on Jan1,2096?

Solution: Given the year 2096 is a leap year.

We know that the leap year ends one day beyond the starting day.

Here the end of the day is given. We have to find the start of the day.

In leap year starting day will be one day before the end of the day.

∴ The day on Jan 1, 2096, will be Sunday.

EXERCISE

1. If today is Friday after 77777 days the day will be?
2. If today is Saturday then 93 days back what was the day?
3. Find the number of odd days in 1886 years.
4. What was the day when India Won the First cricket world cup on 25 June 1983?
5. What was the day when India Got Independence on August 15th, 1947, and what will be the day When India Completes 100 years of Independence on August 15th, 2047?
6. It was Tuesday on January 12th, 1773 then what will be the day on September 14th of the same year? (Sunday)
7. If January 1st in 1950 was Sunday, then What was the day on Dec 27th, 1950?

8. The calendar for the year 1982 repeats in which upcoming year?

Answers:

1. Friday 2. Thursday 3. 5 4. Saturday 5. Friday, Thursday 6. Tuesday 7. Wednesday 8. 1993

web resources for Advanced learners:

1. <https://www.tutorialride.com>
2. <https://www.careerride.com>
3. <https://youtu.be/oxc4G14nyUY>



UNIT –II

QUANTITATIVE APTITUDE

Averages
Ratio and Proportion
Problems on Ages
Time, Distance and Speed-1
Time, Distance and Speed-2

&

BUSINESS COMPUTATIONS

Percentages
Profit & Loss
Partnership
Simple Interest
Compound Interest

2.1 AVERAGES

LEARNING OUTCOMES:

After the completion of this topic the student will be able to

- Understand the concept of averages
- Learn different types of Averages and their properties
- Find the suitable average for the given problems.

Introduction:

To represent the entire group by a single value in a simple and effective way we use the concept of average. This chapter enables the student how to find the average marks, height, Income, age, speed, velocity, etc. by using the suitable average.

Natural numbers: The numbers which are used for counting and ordering are called Natural numbers. They are denoted by N .

Set of Natural numbers $N = \{1, 2, 3, \dots\}$.

Prime numbers: The numbers which are divisible by 1 and themselves are called prime numbers.

Examples: 2, 3, 5, 7, 11, 13, 17, 19, etc..

Odd numbers: The integers which leave the remainder of 1 when divided by 2 are called odd numbers.

Examples: 1,3,5,7,9,11,13,----

Even numbers: The integers which leave the remainder 0 when divided by 2 are called even numbers.

Examples: 0,2,4,6,8, — — — —

Note: Multiples of 2: 2,4,6,8, — — — —

Multiples of 3: 3,6,9,12, — — — — —

Multiples of x : $x, 2x, 3x, — — — — —$

Average:

The Average is defined as the ratio of the sum(total) of all the observations(numbers) to the number of observations(numbers). It is also called the Arithmetic mean (A.M) of the observations.

$$\begin{aligned}\text{Arithmetic mean } \bar{x} &= \frac{\text{Sum of the observations}}{\text{Total number of observations}} \\ &= \frac{\sum_{i=1}^n x_i}{n} = \frac{x_1 + x_2 + \dots + x_n}{n}\end{aligned}$$

Arithmetic mean is used to find the average income, average height, average rainfall, etc.

Weighted Arithmetic Mean or Pooled mean: If the weights of the observations $x_1, x_2, x_3, \dots, x_n$, are $w_1, w_2, w_3, \dots, w_n$, then the weighted Arithmetic mean is

$$\bar{x} = \frac{w_1x_1 + w_2x_2 + \dots + w_nx_n}{w_1 + w_2 + w_3, \dots + w_n}$$

Harmonic Mean: To find average speed and velocity we use Harmonic mean.

The harmonic mean of the observations $x_1, x_2, x_3, \dots, x_n$, is

$$\frac{n}{\sum \frac{1}{x_i}}$$

For example, if a, and b are any two observations then the

Harmonic mean between a and b $\frac{2}{\frac{1}{a} + \frac{1}{b}} = \frac{2ab}{a+b}$

Note: The Harmonic mean is not defined if any one of the observations is zero.

Table 1:

Average	Formula
Arithmetic mean	$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n}$
Weighted Arithmetic mean	$\bar{x} = \frac{w_1x_1 + w_2x_2 + \dots + w_nx_n}{w_1 + w_2 + w_3, \dots + w_n}$

Harmonic mean	$\frac{n}{\sum \frac{1}{x_i}}$
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Formulae:

1. Sum of first 'n' natural numbers = $\frac{n(n+1)}{2}$
2. Average of n natural numbers

$$= \frac{\text{Sum of the } n \text{ natural numbers}}{\text{Total number of natural numbers}} = \frac{\frac{n(n+1)}{2}}{n} = \frac{(n+1)}{2}$$
3. Sum of squares of first n natural numbers: =

$$\frac{n(n+1)(2n+1)}{6}$$
4. Average of squares of first n natural numbers: =

$$\frac{(n+1)(2n+1)}{6}$$
5. Sum of cubes of first n natural numbers: = $\frac{n^2(n+1)^2}{4}$
6. Average of cubes of first n natural numbers: =

$$\frac{n(n+1)^2}{4}$$
7. Sum of first n odd numbers = n^2
8. Average of first n odd numbers = n
9. Sum of first n even numbers = $n(n + 1)$
10. Average of first n even numbers = $n + 1$

Table 2:

Average of	Formula
First n natural numbers	$\frac{(n+1)}{2}$
Squares of first n natural numbers	$\frac{(n+1)(2n+1)}{6}$
Cubes of first n natural numbers	$\frac{n(n+1)^2}{4}$
First n odd numbers	n
First n even numbers	$n+1$

WORKED OUT PROBLEMS

1. (a). Find the average of the first 89 natural numbers.

Solution: The average of first n natural numbers $\frac{(n+1)}{2} =$

$$\frac{(89+1)}{2} = \frac{90}{2} = 45$$

(b). Find the average of first 100 natural numbers.

Solution: The average of first n natural numbers $\frac{(n+1)}{2} =$

$$\frac{(100+1)}{2} = \frac{101}{2} = 50.5$$

(c). Find the average of first 1608 natural numbers.

Solution: The average of first n natural numbers $\frac{(n+1)}{2} =$

$$\frac{(1608+1)}{2} = \frac{1609}{2} = 804.5$$

2. Find the average of the first 55 odd numbers.

Solution: Here $n = 55$

Average of first n odd numbers = n

$\Rightarrow$ Average of first 55 odd numbers = 55

Check:

$$\frac{2+4+6+8+10+12+14+16+18+20+22+24+26+28+30+32+34+36+38+40}{20}$$

$$= \frac{420}{20} = 21$$

3. Find the average of all the prime numbers from 17 to 31.

Solution: Prime numbers from 17 to 31:

17, 19, 23, 29, 31.

$$\text{The average} = \frac{17+19+23+29+31}{5} = \frac{119}{5} = 23.8$$

4. Find the average sum of squares of the first 199 natural numbers.

Solution: The formula for finding the average of the sum of the squares of first n natural

$$\text{numbers} = \frac{(n+1)(2n+1)}{6}$$

$$\begin{aligned}
 &= \frac{(199+1)(2 \times 199 + 1)}{6} \\
 &= \frac{(200)(399)}{6} \\
 &= 13,300
 \end{aligned}$$

- 5. Find the average of the sum of cubes of the first 78 natural numbers.**

Sol: The formula for finding the average of the sum of cubes of first n natural

$$\begin{aligned}
 \text{numbers} &= \frac{n(n+1)^2}{4} \\
 &= \frac{78(78+1)^2}{4} \\
 &= \frac{78(79)^2}{4} \\
 &= \frac{78 \times 6241}{4} \\
 &= 121699.5
 \end{aligned}$$

- 6. If the average score of 4 batsmen in cricket at Bangalore is 85 runs, 6 batsmen at Delhi is 76 runs, and 5 batsmen at Mumbai is 70 runs. Then find the average runs scored by all batsmen?**

Solution: Here $x_1 = 85$; $x_2 = 76$; $x_3 = 70$; $w_1=4$;
 $w_2=6$; $w_3=5$;

$$\begin{aligned}
 \bar{x} &= \frac{w_1x_1 + w_2x_2 + \dots + w_nx_n}{w_1 + w_2 + w_3, \dots + w_n} \\
 &= \frac{4(85) + 6(76) + 5(70)}{4 + 6 + 5} \\
 &= \frac{340 + 456 + 350}{15} \\
 &= \frac{1146}{15} = 76.4
 \end{aligned}$$

7. Find the Harmonic mean of 4,8,10.

Solution: Here $x_1 = 4, x_2 = 8, x_3 = 10$ and $n = 3$.

$$\text{Harmonic mean} = \frac{n}{\sum \frac{1}{x_i}} = \frac{3}{\frac{1}{4} + \frac{1}{8} + \frac{1}{10}} = 5.2174$$

8. If the average of 15,29,35, x , 52 is twice the average of 9, 14, x , 18,20 then find the value of x .

Solution: The average of 15,19,25, x , 32 =

$$\frac{15+29+35+x+52}{5} = \frac{131+x}{5}$$

The average of 9,14, x , 18,20 =

$$\frac{9+14+x+18+20}{5} = \frac{61+x}{5}$$

$$\Rightarrow \text{According to the given problem } \frac{131+x}{5} = 2\left(\frac{61+x}{5}\right)$$

$$\Rightarrow \frac{131+x}{5} = \frac{122+2x}{5}$$

$$\Rightarrow 131 + x = 122 + 2x$$

$$\Rightarrow 131 - 122 = 2x - x \Rightarrow 9 = x$$

$$\Rightarrow x = 9$$

12. The average height of a cricket team is 177cm . If one of the cricketer's heights is misread as 186cm instead of 182cm then what is the correct average?

Solution: Given the Average height of 11 cricketers

$$= 177\text{cm}. (\because \text{Team consists of 11 members})$$

$$\Rightarrow \text{Total height of Cricket team} = 177 \times 11 = 1947\text{cm}.$$

Given that one of the cricketer's heights was misread as 186 instead of 182. So subtract 186 and add 182 to the total height.

Corrected total height of the team

$$= 1947 - 186 + 182 = 1943.$$

$$\therefore \text{The new average} = \frac{1943}{11} = 176.63.$$

13. The average consumption of electricity for 100 houses in a certain village is $90\text{units}(kwh)$. If 15 houses are switched to a Solar electric system then the average consumption of electricity became $77\text{ units}(Kwh)$. Then what was the average consumption of that 15 houses?

Solution: The total consumption of electricity in 100 houses

$$= 100 \times 90\text{units} = 9000\text{ units}(kwh)$$

Given that 15 houses are switched to Solar electric system and the average consumption of 85(100 – 15) houses became 77units

The total consumption of electricity of 85 houses

$$= 85 \times 77 \text{units} = 6545 \text{ units(kwh)}$$

∴ The total consumption of 15 houses

$$= 9000 - 6545 = 2455$$

The average consumption of 15 houses $= \frac{2455}{15} = 163.66$

14. The average weight of a class of 20 students is 40.5 kg.

If a student weighing 42kg has left the class and another student of weight 35 kg has joined the same class then the average weight of the class is?

Solution: Given the average weight of a class of 20 students is 40.5 kg.

We know that Average weight $= \frac{\text{Total weight of the class}}{\text{No.of students}}$

$$\Rightarrow \text{Total weight of the class} = (\text{No.of students})(\text{Average weight}) = 20 \times 40.5 \text{kg} = 810 \text{ kg}$$

Here a student of weight 42 kg left the class so we have to subtract 42 kg from the total weight and a student of weight 35 kg joined so we have to add 35 kg to the total weight.

∴ Revised total weight of the class

$$= 810 \text{ kg} - 42 \text{ kg} + 35 \text{ kg} = 803 \text{ kg}$$

$$\therefore \text{The average} = \frac{803 \text{ kg}}{20} = 40.15 \text{ kg}$$

15. The average of five consecutive odd numbers is

55. Find the second largest number.

Solution: Let the five consecutive odd numbers be

$$x, x + 2, x + 4, x + 6, x + 8.$$

Given an Average = 55

$$\Rightarrow \frac{x + (x+2) + (x+4) + (x+6) + (x+8)}{5} = 55$$

$$\Rightarrow \frac{5x+20}{5} = 55$$

$$\Rightarrow 5x + 20 = 55 \times 5 = 275$$

$$\Rightarrow 5x = 275 - 20$$

$$\Rightarrow 5x = 255$$

$$\Rightarrow x = \frac{255}{5} = 51$$

∴ The five consecutive odd numbers are: $x, x +$

$$2, x + 4, x + 6, x + 8.$$

Substituting 'x' we get 51, 53, 55, 57, 59.

∴ The five consecutive odd numbers are:

$$51, 53, 55, 57, 59.$$

In the problem, they have asked to find the second largest odd number.

The first largest odd number is 59

The second largest odd number is: 57

Note: In the above problem, the smallest odd number is 51.

EXERCISE

1. The average age of $x + 2, 2x, 3 - x, 15$ is 40. then find the value of x .
2. The average of 20 observations is ' x '. Find the average if
 - (a) All the observations are decreased by 7
 - (b) All the observations are increased by 10
 - (c) All the observations are multiplied by 6
3. The average marks of Sameer, Sandy, and Srujan in Mathematics is 87. If the average marks of Sandy and Srujan is 90. Then find the marks of Sameer.
4. The average salary of 10 teachers increased by 500 when two of them with salary of 35,000 and 38,000 transferred to another place and two newly joined. What was the average salary of newly joined teachers?

5. The average of 20 students in a class is 18 years. The average age of the first 12 students is 17 years and the last 7 students is 20 years. Then find the average age of the 13th student.
6. The average of 4 consecutive odd numbers is 10. Then find the product of the smallest and largest numbers.
7. If a student goes to college from his/her home by bicycle at the speed of 9 km per hour and comes back home at the speed of 7 km per hour. Then find the average speed covered by the student.
8. 50 employees are allotted to 3 cities namely A, B, and C. If the average salary of 15 employees allotted to City A is 25,000, 19 employees allotted to city B is 28,000, and 16 employees allotted to City C is 26,000. Then find the average salary of all the employees.
9. The average cost of 15 air coolers is 8000. If 5 air coolers were added then average prices decreased by 500. Then find the average cost of new air coolers.
10. The average age of P, Q, R and S is 26 years. If the average age of Q and R is 24 years, the average age of R and S is 27 years, and the average age of Q and S is 25 years then find the ages of P, Q, R and S

Answers:

1. 70	2 $(a)x - 7$, $(b)x + 10$. $(c)6x$	3.81	4.39,000	5. 16 years
6. 91	7. 7.875	8. 26460	9. 6000	10. 28,22,26 and 28.



Web resources:

1. <https://www.tutorialride.com>
2. <https://www.careerride.com>
3. <https://youtu.be/rhSxQ4ieAYc>

2.2 RATIO AND PROPORTION

LEARNING OBJECTIVES:

After the completion of the topic, the student will be able to

- Understand the concepts of ratio and proportions.
- Learn the properties of ratios and proportions.
- Compute the problems on ratio& proportions

Introduction:

The concepts of ratio and proportions are used in business, preparing alloys, construction, painting, car transmissions, cooking, etc. A comparison of two quantities of the same kind (expressed in the same units) is called the ratio and when two ratios are equal, then the ratios are said to be in proportion.

Definitions

Ratio: A Comparison between two quantities of the same units expressed in the form of a fraction is called the ratio. For example, if P and Q are two quantities of the same units then $\frac{P}{Q} = P:Q$ is called the ratio between P and Q .

Here the first term P is called the **antecedent**, and the second term Q is called the **consequent**.

Note:- The ratio of 2 quantities remains unchanged if we multiply or divide the quantities by the same number.

For example (i) $5:3 = 15:9 = 30:18 =$

$120:72$ (ii) $16:40 = 8:20 = 4:10 = 2:5$

Duplicate ratio: If $a:b$ is the ratio of a and b then $a^2:b^2$ is called the Duplicate ratio of a and b .

Example: Duplicate ratio of $7:9 = 7^2:9^2 = 49:81$

Sub-duplicate ratio: If $a:b$ is the ratio of a and b then $\sqrt{a}:\sqrt{b}$ is called the Sub Duplicate ratio of a and b

Example: Sub Duplicate ratio of $64:225 = \sqrt{64}:\sqrt{225} = 8:15$

Triplicate ratio: If $a:b$ is the ratio of a and b then $a^3:b^3$ is called the Triplicate ratio of a and b

Example: Triplicate ratio of $2:6 = 2^3:6^3 = 8:216$

Sub- Triplicate ratio: If $a:b$ is the ratio of a and b then

$\sqrt[3]{a}:\sqrt[3]{b} = a^{\frac{1}{3}}:b^{\frac{1}{3}}$ is called the Triplicate ratio of a and b

Example: Sub-triplicate ratio of $343:729 = \sqrt[3]{343}:$

$$\sqrt[3]{729} = (343)^{\frac{1}{3}}:(729)^{\frac{1}{3}} = 7:9$$

Inverse ratio: If $a:b$ is the ratio then $\frac{1}{a}:\frac{1}{b}$ is called the inverse ratio of a and b .

Example: Inverse ratio of 4:9 is $\frac{1}{4}:\frac{1}{9}$

Ratio	Duplicate ratio	Sub duplicate ratio	Triplicate ratio	Sub triplicate ratio	Inverse or reciprocal ratio
$a:b$	$a^2:b^2$	$\sqrt{a}:\sqrt{b}$	$a^3:b^3$	$\sqrt[3]{a}:\sqrt[3]{b}$	$\frac{1}{a}:\frac{1}{b}$ or $b:a$

Compound ratio: The ratio of the product of the antecedents to that of the consequents of two or more ratios is called the compound ratio. For example, If $a:b$ and $c:d$ are two ratios then the compound ratio of the given ratios is $ac:bd$

Note: The compound ratio of $b:c$ and $e:f$ is $ace:bdf$

Comparison of ratios: To compare the ratios $\frac{a}{b}$ and $\frac{c}{d}$ find ad and bc .

- If $ad > bc$ then $\frac{a}{b} > \frac{c}{d}$.
- If $ad < bc$ then $\frac{a}{b} < \frac{c}{d}$.

- If $ad = bc$ then $\frac{a}{b} = \frac{c}{d}$.

Proportion: If two ratios are equal i.e., if, $\frac{a}{b} = \frac{c}{d}$, then a, b, c and d are said to be in proportion. We write it as $a : b :: c : d$. Here first and last terms a and d are called extremes, and middle terms b and c are called means. Here a is called first proportional, b is called second proportional, c is called third proportional and d is called fourth proportional.

Note: $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$.

Examples: 1. If $\frac{4}{9} = \frac{12}{27}$, we write $4 : 9 :: 12 : 27$.

Note:

- Third proportional of a, b is $\frac{b^2}{a}$.
- Fourth proportional of a, b, c is $\frac{bc}{a}$.
- Mean proportional of a, b is $\sqrt{ab}$.

Properties of proportions:

$$1. \quad \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{b}{a} = \frac{d}{c}. \quad (\text{Invertendo})$$

Example: If $\frac{5}{15} = \frac{7}{21}$ then $\frac{15}{5} = \frac{21}{7}$

$$2. \quad \frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a}{c} = \frac{b}{d}. \quad (\text{Alternendo})$$

Example: If $\frac{4}{28} = \frac{2}{14}$ then $\frac{4}{2} = \frac{28}{14}$

3. $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{b} = \frac{c+d}{d}$. (Componendo)

Example: If $\frac{6}{5} = \frac{30}{25}$ then $\frac{6+5}{5} = \frac{30+25}{25}$ i.e., $\frac{11}{5} = \frac{55}{25}$

4. $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)

Example: If $\frac{6}{5} = \frac{30}{25}$ then $\frac{6-5}{5} = \frac{30-25}{25}$ i.e., $\frac{1}{5} = \frac{5}{25}$

5. $\frac{a}{b} = \frac{c}{d} \Rightarrow \frac{a+b}{a-b} = \frac{c+d}{c-d}$. (Componendo and Dividendo)

Example: If $\frac{6}{5} = \frac{30}{25}$ then $\frac{6+5}{6-5} = \frac{30+25}{30-25}$ i.e., $\frac{11}{1} = \frac{55}{5}$

WORKED OUT PROBLEMS

1. Find the fourth proportional to the numbers 5, 7, 35.

Solution: Let $a = 5$, $b = 7$, and $c = 35$. We have to find the value of d .

We know that $\frac{a}{b} = \frac{c}{d} \Rightarrow ad = bc$.

$$\frac{5}{7} = \frac{35}{d} \Rightarrow 5d = 7 \times 35 \Rightarrow d = \frac{7 \times 35}{5} = 49.$$

2. Find the third proportional to the numbers 9, 6.

Solution: Third proportional of 9, 6 is $\frac{6^2}{9} = \frac{36}{9} = 4$.

3. Find the mean proportional between 44 and 11.

Solution: Let $a = 44$, and $b = 11$.

Mean proportional of a, b is $\sqrt{ab}$

$\Rightarrow$ The mean proportional of 44, 11 is $\sqrt{44 \times 11} = \sqrt{484} = 22$.

4. Which is greater $\frac{11}{18}$ or $\frac{13}{22}$?

Solution: Let $\frac{a}{b} = \frac{11}{18}$ and $\frac{c}{d} = \frac{13}{22}$. $\Rightarrow ad = 11 \times 22 = 242$; $bc = 18 \times 13 = 234$.

Here $ad > bc$. So $\frac{a}{b} > \frac{c}{d} \Rightarrow \frac{11}{18} > \frac{13}{22}$.

5. Which ratio is least 14:17 or 18:21?

Solution: Let $a:b = 14:17 \Rightarrow \frac{a}{b} = \frac{14}{17}$

$c:d = 18:21 \Rightarrow \frac{c}{d} = \frac{18}{21}$.

$\Rightarrow ad = 14 \times 21 = 294$; $bc = 17 \times 18 = 306$

Here $ad < bc$. So $\frac{a}{b} < \frac{c}{d} \Rightarrow \frac{14}{17} < \frac{18}{21}$.

$\therefore$ The least ratio is 14:17.

6. Two numbers are in the ratio 5:7. If the difference between them is 48 then find the sum of the numbers.

Solution: Let the numbers be $a = 5x$ and $b = 7x$.

Given that the difference between the numbers = 48

$$\Rightarrow 7x - 5x = 48$$

$$\Rightarrow 2x = 48$$

$$\Rightarrow x = 24.$$

$\therefore$ The numbers are $5x, 7x = 5 \times 24, 7 \times 24 = 120, 168$.

$\Rightarrow$ The sum of the numbers $= 120 + 168 = 288$.

7. **If $A:B = 2:3$ and $B:C = 4:9$ then find $A:B:C$.**

Solution: Given $A:B = 2:3$ and $B:C = 4:9$ Multiply the ratio $A:B$ with 4 ($\because B$ in $B:C$ is 4) and the ratio $B:C$ with 3 ($\because B$ in $A:B$ is 3)

Then $A:B = 8:12$ and $B:C = 12:27$.

$\Rightarrow A:B:C = 8:12:27$

8. **If, $A:B = 3:5$, $B:C = 2:7$ and $C:D = 4:9$ then find $A:B:C:D$.**

Solution: Given $A:B = 3:5$, $B:C = 2:7$ and $C:D = 4:9$.

multiply the ratio $A:B$ with 2 and $B:C$ with 5.

Now $A:B = 6:10$ and $B:C = 10:35$.

$\therefore A:B:C = 6:10:35$. Given $C:D = 4:9$

Multiply $A:B:C$ with 4 ($\because C$ in $C:D$ is 4) and $C:D$ with 35 ($\because C$ in $A:B:C$ is 35)

Then $A:B:C = 24:40:140$ and $C:D = 140:315$

$\therefore A:B:C:D = 24:40:140:315$

9. If $4A = 5B = 8C$ then $A : B : C$?

Sol: Let $4A = 5B = 8C = k$.

$$4A = k \Rightarrow A = \frac{k}{4}; 5B = k \Rightarrow B = \frac{k}{5}; 8C = k \Rightarrow C = \frac{k}{8};$$

$$\therefore A : B : C = \frac{k}{4} : \frac{k}{5} : \frac{k}{8} = \frac{1}{4} : \frac{1}{5} : \frac{1}{8} = 10 : 8 : 5$$

($\because$ L.C.M of 4,5 and 8=40, multiply each fraction with 40

$$\text{i.e., } \frac{1}{4}(40) : \frac{1}{5}(40) : \frac{1}{8}(40) = 10 : 8 : 5.)$$

10. The salaries of the employees A, B and C are in the ratio of 7: 9: 11. If the salaries are hiked by 15%, 10% and 5% respectively, then find the new ratio of their salaries.

Solution: Let the salaries of A, B and C be $7x, 9x$, and $11x$ respectively.

If the salary of A hiked by 15% then the new salary will be

$$7x + 7x \times \frac{15}{100} = \frac{700x + 105x}{100} = \frac{805x}{100}.$$

If the salary of B hiked by 10% then the new salary will be

$$9x + 9x \times \frac{10}{100} = \frac{900x + 90x}{100} = \frac{990x}{100}$$

If the salary of C hiked by 5% then the new salary will be

$$11x + 11x \times \frac{5}{100} = \frac{1100x + 55x}{100} = \frac{1155x}{100}$$

$\therefore$ The new ratios will be

$$\frac{805x}{100} : \frac{990x}{100} : \frac{1155x}{100} = 805x : 990x : 1155x =$$

$$805 : 990 : 1155 = 161 : 198 : 231$$

11. Four numbers in the ratio 2:3:5:6 such that the sum of the squares of the numbers is 666. Find the numbers.

Solution: Let the four numbers be $2x, 3x, 5x, 6x$.

Given: The sum of the squares of the numbers is 666.

$$\Rightarrow (2x)^2 + (3x)^2 + (5x)^2 + (6x)^2 = 666.$$

$$\Rightarrow 4x^2 + 9x^2 + 25x^2 + 36x^2 = 666.$$

$$\Rightarrow 74x^2 = 666.$$

$$\Rightarrow x^2 = \frac{666}{74}.$$

$$\Rightarrow x^2 = 9.$$

$$\Rightarrow x = \pm 3$$

The four numbers are $2x, 3x, 5x, 6x = 6, 9, 15, 18$ or $-6, -9, -15, -18$.

12. Two numbers are in the ratio 3: 7. If each number is increased by 9 then ratio becomes 3:5. Then find the sum of cubes of the numbers.

Solution: Let the numbers be $3x, 7x$.

If each number is increased by 9 the ratio becomes

$$3: 5 \Rightarrow 3x + 9: 7x + 9 = 3: 5$$

$$\Rightarrow \frac{3x+9}{7x+9} = \frac{3}{5}$$

$$\Rightarrow 5(3x + 9) = 3(7x + 9)$$

$$\Rightarrow 15x + 45 = 21x + 27$$

$$\Rightarrow 45 - 27 = 21x - 15x$$

$$\Rightarrow 18 = 6x$$

$$\Rightarrow x = 3.$$

13. A box contains Rs. 5, Rs. 10 and Rs. 20 notes in the ratio 7: 5: 11. If the total amount in the box is Rs. 9,150 then find the number of notes of each type.

Solution: Let the number of Rs. 5, Rs. 10 and Rs. 20 notes be $7x$, $5x$ and $11x$ respectively.

According to the problem the total amount in the box

$$= \text{Rs. } 9,150.$$

$$\Rightarrow 5(7x) + 10(5x) + 20(11x) = \text{Rs. } 9,150.$$

$$\Rightarrow 305x = \text{Rs. } 9,150.$$

$$\Rightarrow x = \frac{\text{Rs. } 9,150}{305} = 30.$$

$$\therefore \text{The number of Rs. 5 notes} = 7x = 7 \times 30 = 210.$$

$$\text{The number of Rs. 10 notes} = 5x = 5 \times 30 = 150.$$

$$\text{The number of Rs. 20 notes} = 11x = 11 \times 30 = 330.$$

Check: $210(5) + 150(10) + 330(20) = 1050 + 1500 + 6600 = 9150$.

14. 24 liters of Mango juice contain a mixture of mango pulp, and water in the ratio 5: 3 How much additional water should be added to the juice to get the ratio 5:4

Solution: The proportion of mango pulp in 30 liters of juice $= \frac{5}{5+3} (24) = \frac{5}{8} \times 24 = 15 \text{ litres}$.

The proportion of water in 30 liters of juice $= \frac{3}{5+3} (24) = \frac{3}{8} \times 24 = 9 \text{ litres}$.

Let x liters of water added to the juice then

$$15: (9 + x) = 5: 4.$$

$$\Rightarrow 15 \times 4 = 5(9 + x).$$

$$\Rightarrow 60 = 45 + 5x.$$

$$\Rightarrow 60 - 45 = 5x.$$

$$\Rightarrow 5x = 15 \Rightarrow x = 3.$$

$\therefore$ 3 Liters of water should be added to the juice to get the ratio 5: 4.

Check: $15: (9 + 3) = 15: 12 = 5: 4$

15. A sum of Rs. 5000 is divided among Mahesh, Rajesh, and Suresh in such a way that Rajesh gets Rs.600 more than Mahesh, and Suresh gets Rs.400 less than what Rajesh gets. Find the ratio of their shares.

Solution: Let the share of Mahesh be ' x '.

According to the problem Rajesh gets Rs.600 more than Suresh.

$\Rightarrow$ The share of Rajesh is ' $x+600$ '.

Suresh gets Rs.400 less than Rajesh.

$\Rightarrow$ The share of Suresh is ' $x + 600 - 400$ ' = $x + 200$.

The total sum is Rs.5000. $\Rightarrow x + x + 600 + x + 200 = 5000$

$$\Rightarrow 3x + 800 = 5000$$

$$\Rightarrow 3x = 4200$$

$$\Rightarrow x = \frac{4200}{3} = 1400.$$

$\therefore$ The share of Mahesh = $x = 1400$.

The share of Rajesh = $x + 600 = 2000$.

The share of Suresh = $x + 200 = 1600$.

∴ The ratio of the shares = 1400:2000:1600

$$= 14:20:16 = 7:10:8$$

EXERCISE

1. Find the third proportional to the numbers 4.5 and 6.5.
2. If $\frac{a}{5} = \frac{b}{7} = \frac{c}{11}$ then the value of $\frac{a+b+c}{a}$?
3. The ratio between the ages of Arjun and Pranav is 2:3. If the sum of their ages is 50 then find the age of Pranav.
4. A bag contains 50 paise, 20 paise, and 5 paise coins in the ratio 2:6:15. The total amount in the bag is Rs 590. Find the number of coins of each type.
5. The salaries of the 3 employees Radha, Sunitha, and Vasavi are in the ratio 3:6:8. If the increments of the employees are 5%, 10%, and 15% respectively then find the new ratio.
6. If 20% of x is twice as 15% of y then find the ratio of x and y .

7. In a college the ratio of the number of teaching and non-teaching staff is 12:7. If there are 152 staff then find the number of non-teaching staff.
8. The proportion of Gold and copper in an ornament is 9:2. Find the amount of copper required for 23 gm of Gold.

Answers

1. 9.388 2. $\frac{23}{5}$ 3. 30 4.
400,1200,3000. 5. 63:132:184. 6. 3:2
7. 56 8. 5.111



web resources:

1. <https://www.tutorialride.com>
2. <https://www.careerride.com>
3. <https://youtu.be/jfoJBivWlnQ>

2.3 : PROBLEMS ON AGES

Learning Outcomes :

After completion of the topic student can able to

1. Identify the different concepts of age.
2. Identify the inequalities.
3. Improve analytical ability to solve quantitative aptitude questions.
4. Understand the logic and solve the problems.
5. Apply and enhances problem solving skill,

This topic is one of the parts in Aptitude. The number of years something has been alive or span of years during which something event occurred is known as “Age”. Problems on ages are generally contains information of two or more persons and a relation between their ages in present or past. Generally, age can be measured in Years, Months. Age categories are,

Children 0-14 years

Youth 15-24 years

Adults 25-64 years

Seniors Above 65 years.

Basic concepts of age:

1. **After the year:** If you are assuming the present age to be x , then the age after “ n ” years will be $(x + n)$ years.
i. e, If we have to find the age of a person after “ n ”

years then we will have to add “**n**” years to the present age.

Example-1: Present age of pradyumna is 13 years . What was his age after 15 years?

Solution: Present age of Pradyumna = 13 years

Pradyumna’s age after $n = 15$ years is $(13+n) = 13 + 15$
 $= 28$ years

Example-2: The present age of suri is 6 times of vani’s age. Vani’s present age is 10 years less than suri’s age . Find the present age of suri ?

Solution: Let the present age of Vani = x years

Present age of suri = $6x$ years

Therefore $6x - x = 10 \Rightarrow 5x = 10 \Rightarrow x = \frac{10}{5} = 2$

Therefore present age of suri = $6 \times 2 = 12$ years

2. **Before the year :** If you are assuming the present age to be **x**, then the age before “**n**” years will be **(x - n)** years. .
i. e If we have to find the age of a person before “**n**” years then we will have to subtract “**n**” years from the present age.

Example-1: Present age of Rohan is 28 years . What was his age Before **9** years?

Solution: Present age of Rohan is = 28 years

Rohan's age before $n = 9$ years is $(28 - n) = 28 - 9$
 $= 19$ years.

Example-2: Find the present age of a person if 15 years before his age will be 4 times of what he was 15 years ago.?

Solution : Let the present age = x years

$$(x+15) = (x-15) 4$$

$$X+15 = 4x - 60 \Rightarrow 3x = 75$$

Therefore $x = 25$ years.

3. Ratio of ages : If present age of A and B is X years and Y years respectively then the ratio of their ages after

$$\text{"n"} \text{ years} = \frac{x+n}{y+n} .$$

If present age of A and B is X years and Y years respectively then the ratio of their ages before "n" years

$$= \frac{x-n}{y-n} .$$

Example-1: The present age of Raja and Ramesh is in the ratio 3:4 , 5 years before their ages are in the ratio of 2:3. What is the present age of Raja?

Solution: Let the present age of Raja = $3x$

Let the present age of Ramesh = $4x$

5 years back Raja's age = $3x-5$ years
5 years back Ramesh's age = $4x-5$ years

Ratio of their ages before 5 years is $\frac{3x-5}{4x-5} = \frac{2}{3}$

$$3(3x-5) = 2(4x-5) \Rightarrow 9x-15 = 8x-10$$

$$\Rightarrow 9x-8x = 15-10 \Rightarrow x = 5$$

Therefore the present age of Raja = 15 years.

Example-2 : The respective ratio of the present ages of the daughter and mother is 1:2, after 6 years the ratio of their ages are in the ratio of 11:20. What is the present age of Mother?

Solution: Let the present age of mother = $2x$

Let the present age of Daughter = x

After 6 years mother's age = $2x+6$ years

After 6 years daughter's age = $x+6$ years

Ratio of their ages after 6 years is $\frac{x+6}{2x+6} = \frac{11}{20}$

$$20(x+6) = 11(2x+6) \Rightarrow 20x+120 = 22x+66$$

$$\Rightarrow 22x-20x = 120-66 \Rightarrow 2x = 54 \Rightarrow x = 27$$

Therefore the present age of mother = 54 years.

EXERCISE

1. Rohan's mother is 26 years older than him. The product of their ages after 3 years will be 360 years. What is Rohan's present age?
 a) 8 years b) 7 years c) 14 years d) 16 years
2. The sum of ages of 5 children born at the intervals of 3 years each is 50 years. What is the age of the youngest child?
 a) 8 years b) 10 years c) 4 years d) 6 years
3. Present age of Sandhya is 25 years .what is her age after 10 years ? And before 6 years?
 a) 35 years and 20 years b) 40 years and 19 years c) 35 years and 19 years d) 36 years and 19 years
4. A father said to his son, "I was as old as you are at the present at the time of your birth". If the father's age is 38 years now, the son's age five years back was:
 a) 14 years b) 10 years c) 12 years d) 16 years
5. The ages of Sirish and Suresh are in the ratio of 5:6, After 8 years the ratio of their will be 7:8. What is the difference in their ages?
 a) 5 years b) 8 years c) 6 years d) 4 years
6. A is two years older than B who is twice as old as C. If the total of the ages of A, B and C be 27, then how old is B ?
 a) 8 years b) 10 years c) 4 years d) 6 years

7. Present ages of Seema and Naresh are in the respective ratio of 6:5. After 9 years the ratio of their ages will be 9:8. What is the difference in their ages?
- a) 9 years b) 4 years c) 5 years d) 3 years
8. The respective ratio between the present ages of Durga and Rakesh 6:11. 4 years ago the ratio of their ages was 1:2, What will be Rakesh's age after 5 years?
- a) 48 years b) 49 years c) 40 years d) 46 years
9. The respective ratio between present ages of paddu and Rohan and Vijay are in the ratio of 3:4:5. If the average of their present age is 28 years, then what would be the sum of ages paddu and Rohan?
- a) 59 years b) 54 years c) 55 years d) 53 years
10. The present ages of Venkat and Suresh are in the ratio of 4:5. After 6 years the ratio of their ages will be 6:7. What is the difference between their ages?
- a) 9 years b) 4 years c) 3 years d) 5 years

ANSWERS :

- | | |
|-------------|-------------|
| 1. b | 2. b |
| 3. c | 4. a |
| 5. d | 6. b |

7. d

8. b

9. a

10. c

Web Links :

1. <https://www.indiabix.com>
2. <https://www.toppr.com>
3. <https://www.indiabix.com>
4. <https://leverageedu.com>
5. <https://www.careerride.com/>



2.4. TIME - DISTANCE - SPEED - 1

LEARNING OUTCOMES

After reading this a student will be able to -

- ❖ Understand the terms time, distance, and speed
- ❖ Understand the relation between time, distance and speed
- ❖ Solve problems on time, distance and speed

Introduction:

Motion occurs when a body of any shape changes its position with respect to any external stationary point. When a person travels from city X to city Y, we say that he has moved from city X to city Y. In other words, whenever a body moves with respect to a stationary point, we say that the body has undergone a motion with respect to the starting point. Thus, there must have been some displacement with respect to the stationary point on the ground, for motion to have occurred in the body. The mathematical model that describes motion has three variables, namely time, speed and distance.

Definitions:

Time

Time is the duration in hours, minutes or seconds over which the movement occurs.

Speed

Speed is defined as the rate at which a body or an object travel to cover a certain distance.

Distance

Distance is defined as the length of the path travelled by any object or a person between two places.

Relative speed

Relative speed is defined as the speed of a moving body in relation to other moving body.

Relation between Time Speed and Distance:

The speed of an object is the distance travelled by the object in unit time.

$$1. \text{ Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$2. \text{ Time} = \frac{\text{Distance}}{\text{Speed}}$$

$$3. \text{ Distance} = \text{Speed} \times \text{Time}$$

Units of measurement and conversion of units:

It is to be noted that units of time, speed and distance should be in the same metric system.

- Generally, if the distance is measured in kilometre, we measure time in hours and speed in kilometre per hour and is written as km/hr or kmph.
- If the distance is measured in metre, then time is taken in seconds and speed in metres per second and is written as m/s.

Conversion Factor:

1. From km/hr to m/s

$$1 \text{ km/hr} = \frac{1000 \text{ metres}}{3600 \text{ seconds}} = \frac{5}{18} \text{ m/s}$$

$$x \text{ km/hr} = x \times \frac{5}{18} \text{ m/s}$$

2. From m/s to km/hr

$$1 \text{ m/s} = \frac{\frac{1}{1000} \text{ km}}{\frac{1}{3600} \text{ hours}} = \frac{18}{5} \text{ km/hr}$$

$$x \text{ m/sec} = x \times \frac{18}{5} \text{ km/hr}$$

Problem solving tricks and important formulae:

- If time taken is constant, then the distance travelled is proportional to the speed, i.e. more the speed more the distance travelled in same time or $\frac{D_1}{S_1} = \frac{D_2}{S_2}$
- If the speed is constant, the distance travelled is proportional to the time taken, i.e. more the distance travelled, more the time taken at the same speed or $\frac{D_1}{T_1} = \frac{D_2}{T_2}$
- If the distance travelled is constant, the speed is inversely proportional to the time taken, i.e. more the speed, less the time taken for the same distance travelled

Or $S_1 T_1 = S_2 T_2 = S_3 T_3 = \dots\dots\dots$

- When two bodies A and B are moving with speeds S_1 km/hr and S_2 km/hr respectively, then the relative speed of two bodies is
 - ❖ $(S_1 - S_2)$ km/hr, if they are moving in same direction.
 - ❖ $(S_1 + S_2)$ km/hr, if they are moving in opposite directions.

- If the ratio of speeds of A and B is $S_1 : S_2$ then the ratio of the times taken by them to cover the same distance

$$\text{is } T_1 : T_2 = \frac{1}{S_1} : \frac{1}{S_2} = S_2 : S_1$$

WORKED OUT PROBLEMS

1. Convert 90 km/hr into m/s.

Sol. We know that, $x \text{ km/hr} = x \times \frac{5}{18} \text{ m/s}$

$$\therefore 90 \text{ km/hr} = \left(90 \times \frac{5}{18}\right) \text{ m/s} = 25 \text{ m/s}.$$

2. Convert 20m/s into km/hr.

Sol. We know that, $x \text{ m/sec} = x \times \frac{18}{5} \text{ km/hr}$

$$\therefore 20 \text{ m/s} = \left(20 \times \frac{18}{5}\right) \text{ km/hr} = 72 \text{ km/hr}$$

3. A bus covers 90 km in 2 hrs, then find the speed of the bus.

Sol. We know that, $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

$$\therefore \text{Required speed} = \frac{90}{2} = 45 \text{ km/hr}.$$

4. A train covers a distance of 240 km with a speed of 60 km/hr. How much time is taken by the train to cover this distance?

Sol. Given, speed = 60 km/hr

and distance = 240 km

$$\text{We know that, } Time = \frac{Distance}{Speed} = \frac{240}{60} = 4 \text{ hr}$$

5. A cyclist covers a distance of 660 m in 2 min 45 sec. What is the speed of the cyclist in km/hr?

Sol. We know that, $speed = \frac{Distance}{Time} = \frac{660}{165} = 4 \text{ m/s} = 4 \times \frac{18}{5} = 14.4 \text{ km/hr.}$

$$(\because 2 \text{ min } 45 \text{ sec} = 165 \text{ sec})$$

6. A person walked at a speed of 6 km/hr from a point X to Y and came back from point Y to X at the speed of 4 km/hr. What would be the ratio of the time taken by the person in walking from point X to point Y to that from point Y to X?

Sol. Ratio of speeds = $S_1 : S_2 = 6 : 4 = 3 : 2$

$$\therefore \text{Ratio of time taken} = T_1 : T_2 = S_2 : S_1 = 2 : 3$$

7. Two trains A and B are running in the same direction, at the speed of 60 kmph and 72 kmph respectively. If the length of both the trains is 240 metres, how long will it take for train B to cross A?

Sol. When a train crosses another train, it travels a distance equal to the sum of the lengths of both the trains.

Total distance travelled by the train when it crosses another train = $240 + 240 = 480$ m

Relative speed of the train = $72 \text{ kmph} - 60 \text{ kmph} = 12 \text{ km/hr}$
 $= 12 \times \frac{5}{18} \text{ m/sec} = \frac{10}{3} \text{ m/sec}.$

Hence the required Time = $\frac{\text{Distance}}{\text{Speed}} = \frac{480}{10/3} = 480 \times \frac{3}{10} = 144$ sec.

8. A person covers a distance of $18\frac{2}{5}$ km in 4 hrs. what distance will he cover in 5 hrs?

Sol. Here the speed is kept constant.

Then we have $\frac{D_1}{T_1} = \frac{D_2}{T_2}$

We are given, $D_1 = 18\frac{2}{5}$ km = $\frac{92}{5}$ km; $T_1 = 4$ hrs; $T_2 = 5$ hrs.

$$\frac{92/5}{4} = \frac{D_2}{5} \Rightarrow D_2 = \frac{92 \times 5}{5 \times 4} = 23 \text{ km.}$$

$\therefore$ The distance covered by the person in 5 hrs = 23 km.

9. A person covers a certain distance with a speed of 24 km/hr in 11 min. If he wants to cover the same distance in 8 min, what should be his speed?

Sol. Here the distance is kept constant.

Then we know that $S_1 T_1 = S_2 T_2$

We have $S_1 = 24 \text{ km/hr}$; $T_1 = 11 \text{ min} = \frac{11}{60} \text{ hrs}$; $T_2 = 8 \text{ min} = \frac{8}{60} \text{ hrs}$; $S_2 = ?$

$$\therefore 24 \times \frac{11}{60} = S_2 \times \frac{8}{60} \Rightarrow S_2 = \frac{24 \times 11}{8} = 33 \text{ km/hr}$$

Hence the required speed = 33 km/hr.

10. Anil covers a distance of 16 km, while walking at a speed of 4 km/hr. How much distance he would cover in the same time, if he walks at a speed of 5 km/hr?

Sol. Here the time is kept constant.

Then we know that $\frac{D_1}{S_1} = \frac{D_2}{S_2}$

Given, $D_1 = 16 \text{ km}$; $S_1 = 4 \text{ km/hr}$; $S_2 = 5 \text{ km/hr}$; $D_2 = ?$

$$\therefore \frac{16}{4} = \frac{D_2}{5} \Rightarrow D_2 = \frac{16 \times 5}{4} = 20 \text{ km}$$

Hence the required distance = 20 km.

EXERCISE

1. Convert 72 km/hr into m/s.
2. The speed of a bus is 54 km/hr. Find the distance covered by the bus in 5 sec.
3. The speed of a train is 60km/hr. What is the distance in meters covered by the train in 12 minutes?
4. A van can cover a distance of 1260 km in 35 hours. Find its speed in metres per second.
5. Two men start together to walk a certain distance, one at 5 km/hr and another at 4km/hr. The former reaches half an hour before the later. Find the distance.

6. By walking at $\frac{3}{4}$ of his usual speed, a man reaches his office 20 minutes later than his usual time. Find the usual time taken by him to reach his office.
7. Anand covers a distance of $12\frac{2}{5}$ km in 3 hrs. what distance will he cover in 5 hrs?
8. Ramu covers a certain distance with a speed of 18 km/hr in 8 min. If he wants to cover the same distance in 6 min, what should be his speed?

Answers

1. 20 m/s 2. $\frac{3}{40}$ km 3. 1.2 km 4. 10 m/s
5. 10 km 6. 60 min 7. 60.33 km 8. 24 km/hr

Web Resources for advanced learners:

- ❖ <https://byjus.com/govt-exams/speed-time-distance/>
- ❖ <https://www.indiabix.com/aptitude/time-and-distance/>
- ❖ <https://youtu.be/jzNxXm5twx4>

❖ https://youtu.be/KZ_M5RWaP6A



2.5. TIME – DISTANCE - SPEED – 2

LEARNING OUTCOMES

After reading this a student will be able to -

- ❖ Understand the terms time, distance, speed
- ❖ Understand the relation between time, distance and speed
- ❖ Apply the knowledge of relation between time, distance and speed to solve different types of problems.

Problem solving tricks and important formulae:

- If two persons A and B start at the same time in opposite directions from two points and after passing each other they complete the journey in x and y hours respectively, then,
$$A's \text{ speed} : B's \text{ speed} = \sqrt{y} : \sqrt{x}$$

- If two trains of lengths x and y are moving in opposite directions with speeds S_1 km/hr and S_2 km/hr respectively, then the time taken by the trains to cross each other is equal to $\frac{x+y}{s_1+s_2}$ hrs

- If two trains of lengths x and y are moving in the same direction with speeds S_1 km/hr and S_2 km/hr respectively, then the time taken by the trains to cross each other is equal to $\frac{x+y}{s_1-s_2}$ hrs ($s_1 > s_2$)

- If A goes from point X to point Y at S_1 km/hr and comes back from Y to X at S_2 km/hr, then the average speed during the whole journey is given by

$$\text{Average speed} = \frac{2s_1s_2}{s_1+s_2} \text{ km/hr.}$$

- If a person covers three equal distances at the speed of S_1 km/hr, S_2 km/hr and S_3 km/hr respectively, then the average speed during the whole journey is given by

$$\text{Average speed} = \frac{3s_1s_2s_3}{s_1s_2+s_2s_3+s_3s_1} \text{ km/hr.}$$

- When a body covers different distances with different speeds, then the average speed of the body for the whole journey is defined as the total distance covered

by the body divided by the total time taken to cover the distance.

$$\text{i.e } \textit{Average speed} = \frac{\textit{Total distance covered by the body}}{\textit{Total time taken by the body}}$$

Note:

- The distance covered by a train in passing a pole or a standing man or signal post or any other object of negligible length is equal to the length of the train.
- If a train passes a stationary object for example bridge, platform etc. having some length, then the distance covered by train is equal to the sum of the length of train and that particular object which it is passing.

WORKED OUT PROBLEMS

1. A train covers 90 m in passing a pole. What is the length of the train?

Sol. We know that the distance covered by a train in passing a pole or a standing man or signal post or any other object of negligible length is equal to the length of the train.

Here the distance covered by the train = 90 m

$\therefore$ Length of the train = 90 m

2. A 600 m long train passes a platform which is 400 m long. Find the distance covered by the train in passing the platform.

Sol. We know that when a train passes a stationary object for Example bridge, platform etc. having some length, then the distance covered by train is equal to the sum of the length of train and that particular object which it is passing.

Here, length of the train = 600 m

Length of the platform = 400 m.

Hence, the required distance = $(600 + 400)$ m = 1000 m.

3. Ram starts to cycle from point A to B and at the same time Syam starts to cycle from point B to A. After passing each other, they complete their journeys in 9 hrs and 16 hrs, respectively. Find the ratio of speed of Ram to Syam.

Sol. Given, after passing each other,

time taken by Ram to cover certain distance = $T_1 = 9$ hrs.

and time taken by Syam to cover the distance = $T_2 =$
16 hrs.

Ram's speed : Syam's speed = $\sqrt{T_2} : \sqrt{T_1} = \sqrt{16} : \sqrt{9} =$
4 : 3

4. Two trains of lengths 210 m and 150 m are moving in opposite directions at 10 m/s and 8 m/s, respectively. Find the time taken by the trains to cross each other.

Sol. Here, $x = 210$ m ; $y = 150$ m ; $S_1 = 10$ m/s ; $S_2 = 8$ m/s

Given, the trains are moving in opposite directions.

Hence, the required Time = $\frac{x+y}{s_1+s_2}$ sec = $\frac{210+150}{10+8}$ sec =
20 sec

5. Gopal walked from the village to the railway station at the rate of 25 km/hr and walked back at the rate of 4 km/hr. If the whole journey took 5 hours 48 minutes, find the distance of the railway station from the village.

Sol. We know that, average speed = $\left(\frac{2s_1s_2}{s_1+s_2}\right)$ km/hr.

$$= \left(\frac{2 \times 25 \times 4}{25 + 4} \right) km/hr.$$

$$= \frac{200}{29} km/hr.$$

$$\text{Given, total time taken} = 5 \text{ hrs } 48 \text{ min} = 5 \frac{48}{60} \text{ hrs}$$

$$= 5 \frac{4}{5} \text{ hrs.} = \frac{29}{5} \text{ hrs.}$$

$$\text{Distance} = \text{Speed} \times \text{Time} = \left(\frac{200}{29} \times \frac{29}{5} \right) km$$

$$= 40 km.$$

$$\therefore \text{Distance of the railway station from the village}$$

$$= \left(\frac{40}{2} \right) km = 20 km.$$

6. While covering a distance of 24 km, a person noticed that after walking for 1 hour and 40 minutes, the distance covered by him was $\frac{5}{7}$ of the remaining distance. What was his speed in meters per second?

Sol. Total distance = 24 km.

Let the speed = x km/hr.

Distance covered in 1 hr 40 min i.e.
 $1\frac{40}{60} \text{ hrs i.e. } 1\frac{2}{3} \text{ hrs} = \text{Speed} \times \text{Time} = \frac{5x}{3} \text{ km.}$

Remaining distance = $\left(24 - \frac{5x}{3}\right) \text{ km.}$

$$\begin{aligned} \text{Given, } \quad \frac{5x}{3} &= \frac{5}{7} \left(24 - \frac{5x}{3}\right) \Leftrightarrow \frac{5x}{3} \\ &= \frac{5}{7} \left(\frac{72 - 5x}{3}\right) \end{aligned}$$

$$\Leftrightarrow 7x = 72 - 5x \quad \Leftrightarrow 12x = 72$$

$$\Leftrightarrow x = 6 \text{ km/hr.}$$

$$\begin{aligned} \text{Required speed} &= 6 \text{ km/hr} = \left(6 \times \frac{5}{18}\right) \text{ m/s} \\ &= 1\frac{2}{3} \text{ m/s or } 1.67 \text{ m/s.} \end{aligned}$$

7. Kamala covers three equal distances at the speed of 20 km/hr, 15 km/hr and 10 km/hr respectively. Find out her average speed during the whole journey.

Sol. Here, $s_1 = 20 \text{ km/hr}$; $s_2 = 15 \text{ km/hr}$; $s_3 = 10 \text{ km/hr}$.

We know that,

$$\text{average speed} = \frac{3s_1s_2s_3}{s_1s_2 + s_2s_3 + s_3s_1} \text{ km/hr.}$$

$$\begin{aligned}
 &= \frac{3 \times 20 \times 15 \times 10}{20 \times 15 + 15 \times 10 + 10 \times 20} \\
 &= \frac{3 \times 20 \times 15 \times 10}{300 + 150 + 200} \\
 &= \frac{3 \times 20 \times 15 \times 10}{650} \\
 &= 13 \frac{11}{13} \text{ km/hr} = 13.86 \text{ km/hr.}
 \end{aligned}$$

8. Sita covers a distance of 16 km by bus in 30 min. After the boarding the bus, She took rest for 25 min and covers another 20 km by bus in 35 min. Find her average speed for the whole journey.

Sol. Total distance covered = (16 + 20) km = 36 km.

$$\begin{aligned}
 \text{Total time taken} &= (30 + 25 + 35) \text{ min} = 90 \text{ min} = \frac{90}{60} \text{ hrs} \\
 &= \frac{3}{2} \text{ hrs.}
 \end{aligned}$$

We know that, Average speed =

$$\frac{\text{Total distance covered by the body}}{\text{Total time taken by the body}}$$

$$\Rightarrow \text{Average speed} = \frac{36}{3/2} = 36 \times \frac{2}{3} = 24 \text{ km/hr.}$$

$\therefore$ The average speed of Sita for the whole journey = 24 km/hr.

EXERCISE

1. The speeds of three cars are in the ratio of 2 : 3: 5. Find the ratio of the time taken by the above cars to travel the same distance.
2. Anil and Sunil start at the same time to ride from Bangalore to Mysore, 150 km away. Anil travels 5 km/hr slower than Sunil. Sunil reaches Mysore and at once turns back meeting Anil 18 km from Mysore. Find Anil's speed.
3. By walking at $\frac{3}{4}$ of his usual speed, a man reaches his office 20 minutes later than his usual time. Find the usual time taken by him to reach his office.
4. A man covers a certain distance with a speed of 18 km/hr in 8 min. If he wants to cover the same distance in 5 min, what should be his speed.

5. Gowri walked from the village to the bus station at the rate of 20 km/hr and walked back at the rate of 4 km/hr. If the whole journey took 6 hours 48 minutes, find the distance of the bus station from the village.
6. Anand covers a distance of $12\frac{2}{5}$ km in 3 hrs. What distance will he cover in 5 hrs?
7. Suman covers a certain distance at a speed of 70 km/hr by car and he returns back to the starting point riding on a scooter with a speed of 40 km/hr. Find the average speed for the whole journey
8. Avinash covers 30 km at a speed of 10 km/hr by a cycle, 15 km at 5 km/h on foot and another 100 km at 50 km/ hr by bus. Then find the average speed for the whole journey.
9. Sony starts to walk from point A to B and at the same time Rani starts to walk from point B to A. After passing each other, they complete their journeys in 4 hrs and 9 hrs, respectively. Find the ratio of speed of Sony to Rani.

Answers

1. 15 : 10 : 6 2. 18.33 km/hr 3. 60 min.
4. 28.8 km/hr 5. 22.67 km 6. 60.33 km
7. 55 km/hr 8. 18.13 km/hr 9. 3 : 2

Web Resources for advanced learners:

- ❖ <https://byjus.com/govt-exams/speed-time-distance/>
- ❖ <https://www.indiabix.com/aptitude/time-and-distance/>
- ❖ <https://youtu.be/jzNxXm5twx4>
- ❖ https://youtu.be/KZ_M5RWaP6A



2.6 Percentage

LEARNING OUTCOMES

After completion of this topic, students will be able to

- Covert fraction to percentage and percentage to fraction.
- Find percentage of a quantity
- Express one quantity as a fraction of another
- Use percentages in both classroom and every day scenario
- Compare quantities with different denominators
- Solve problems on increase or decrease of percentage
- Identify new effect with increase or decrease of percentage

Introduction:

The term percentage comes from “per cent”, it means “out of a hundred”. The given number is split into hundred equal parts, each part is a percent. The symbol of percentage is “%”. The percentage used in calculations and finding profits easy and faster. The percentage works with the denominator 100.

Convert Fraction into Percentage:

Express a fraction $\frac{x}{y}$ into percentage

$$\text{Required percentage} = \left(\frac{x}{y} \times 100\right)\%$$

Ex: Convert $\frac{3}{8}$ into percentage.

Sol: Required percentage = $\left(\frac{3}{8} \times 100\right)\% = 37.5\%$

Convert Percentage into Fraction:

Express a percentage $x\%$ into fraction

$$\text{Required fraction} = \frac{x}{100}$$

Ex: Express 60% into fraction.

Sol: Required fraction = $\frac{60}{100} = \frac{3}{5}$

Convert Percentage into Decimal:

Express a percentage $x\%$ into decimal

$$\text{Required fraction} = \frac{x}{100} = 0.0x$$

Ex: Express 60% into decimal.

Sol: Required fraction = $\frac{60}{100} = 0.6$

Formula to calculate percent:

If we have to find $x\%$ of y , then

$$x\% \text{ of } y = \frac{x}{100} \times y$$

Expressing one quantity as a percent with respect to other:

To express a quantity as a percent with respect to other quantity we use the following formula

$$\text{Required percentage} = \frac{\text{The quantity to be expressed in percent}}{\text{Other quantity}} \times 100$$

Ex: 60 kg is what percent of 300 kg

$$\text{Sol: Required percentage} = \frac{60}{300} \times 100 = 20\%$$

Percentage increase or decrease of a value:

$$\text{Increase}\% = \frac{\text{Increase value}}{\text{Original value}} \times 100\%$$

$$\text{Decrease}\% = \frac{\text{Decrease value}}{\text{Original value}} \times 100\%$$

WORKED OUT PROBLEMS

Type 1: Simplification

1. 40% of ? = 56

Sol: Given 40% of ? = 56

$$\Rightarrow \frac{40}{100} \times ? = 56$$

$$\Rightarrow ? = \frac{56 \times 100}{40}$$

$$\Rightarrow ? = 140$$

2. Find the value whose 32% of $\frac{5}{8}$ is 2900

Sol: Let x be the required value.

Given that 32% of $\frac{5}{8}$ of x is 2900

$$\Rightarrow \frac{32}{100} \times \frac{5}{8} \times x = 2900$$

$$\Rightarrow x = \frac{2900 \times 8 \times 100}{5 \times 32}$$

$$\Rightarrow x = 14,500$$

3. 20% of 500 – ? of 140 = 89

Sol: Given 20% of 500 – ? of 140 = 89

$$\Rightarrow \frac{20}{100} \times 500 - \frac{?}{100} \times 140 = 89$$

$$\Rightarrow 100 - \frac{?}{10} \times 14 = 89$$

$$\Rightarrow 100 - 89 = \frac{?}{10} \times 14$$

$$\Rightarrow \frac{11 \times 10}{14} = ? \Rightarrow ? = \frac{55}{7}$$

4. 0.8% of 450 ÷ 0.5% of 140 = ?

Sol: Consider 0.8% of 450 ÷ 0.5% of 140

$$= \frac{0.8}{100} \times 450 \div \frac{0.5}{100} \times 140$$

$$= \frac{8}{1000} \times 450 \div \frac{5}{1000} \times 140$$

$$= \frac{18}{5} \div \frac{7}{10}$$

$$= \frac{18}{5} \times \frac{10}{7} = \frac{36}{7}$$

5. The difference between 30% of a value and $\frac{1}{5}$ th of the same value is 3400. Find the 15% of that value.

Sol: Let x be the required value.

Given that difference between 30% of a value and $\frac{1}{5}$ th of the same value is 3400

$$\text{i.e., } 30\% \text{ of } x - \frac{1}{5} \text{ of } x = 3400$$

$$\Rightarrow \frac{30}{100} \times x - \frac{1}{5} \times x = 3400$$

$$\Rightarrow x \left(\frac{3}{10} - \frac{1}{5} \right) = 3400$$

$$\Rightarrow x \left(\frac{3-2}{10} \right) = 3400$$

$$\Rightarrow x \left(\frac{1}{10} \right) = 3400$$

$$\Rightarrow x = 3400 \times 10$$

$$\Rightarrow x = 34000$$

$$\therefore 15\% \text{ of } x = \frac{15}{100} \times 34000 = 5100$$

- 6. In an election, 609000 votes were polled. In that 13% of votes were invalid. If a candidate got 70% of valid votes, calculate the no. of valid votes polled in favour of the candidate.**

Sol: Given that total no. of votes = 609000

$$\begin{aligned}
 \text{Total no. of invalid votes} &= 13\% \text{ of } 609000 \\
 &= \frac{13}{100} \times 609000 \\
 &= 79170 \\
 \text{Total no. of valid votes} &= 609000 - 79170 \\
 &= 529830 \\
 \text{Given that the candidate got} &= 70\% \text{ of votes.} \\
 \therefore \text{No. of valid votes polled} &= 70\% \text{ of } 529830 \\
 &= \frac{70}{100} \times 529830 \\
 &= 370881
 \end{aligned}$$

Type 2: Distribution of sum in different percentages and Comparison of quantities

- 1. A sum of total Rs. 6900 distributed among Sita, Geetha and Deepthi such that Sita receives 20% more than Geetha and Geetha receives 20% less than Deepthi. What is Deepthi's share?**

Sol: Let Deepthi's share = x

Geetha's share = 20% less than Deepthi's share

$$= x - 20\% \text{ of } x$$

$$= x - \frac{20}{100} \times x$$

$$= x - \frac{1}{5}x$$

$$= \frac{5x - x}{5} = \frac{4x}{5}$$

Sita's share = 20% more than Geetha's share

$$= \frac{4x}{5} + 20\% \text{ of } \frac{4x}{5}$$

$$= \frac{4x}{5} + \frac{20}{100} \times \frac{4x}{5}$$

$$= \frac{4x}{5} + \frac{4x}{25}$$

$$= \frac{20x + 4x}{25} = \frac{24x}{25}$$

Given that total amount = 6900

$$\Rightarrow \left(\frac{24x}{25} + \frac{4x}{5} + x \right) = 6900$$

$$\Rightarrow \frac{24x + 20x + 25x}{25} = 6900$$

$$\Rightarrow \frac{69x}{25} = 6900$$

$$\Rightarrow x = \frac{6900 \times 25}{69} = 2500$$

$\therefore$ Deepthi's share = Rs. 2,500

2. Anil spends 45% of his monthly salary on ration, 15% on

house rent, 10% on entertainment, 2% on medicines and saves Rs. 5,600. Calculate his annual income.

Sol: Let Anil's monthly income = x

For ration he spends = 45% of his monthly salary

$$= 45\% \text{ of } x = \frac{45}{100} \times x = \frac{9x}{20}$$

$$\text{For house rent he spends} = 15\% \text{ of } x = \frac{15}{100} \times x = \frac{3x}{20}$$

$$\text{For entertainment he spends} = 10\% \text{ of } x = \frac{10}{100} \times x = \frac{x}{10}$$

$$\text{For medicines he spends} = 2\% \text{ of } x = \frac{2}{100} \times x = \frac{x}{50}$$

His Savings = Rs. 5,600

$$\therefore \frac{9x}{20} + \frac{3x}{20} + \frac{x}{10} + \frac{x}{50} + 5600 = x$$

$$\Rightarrow \frac{45x + 15x + 10x + 2x}{100} + 5600 = x$$

$$\Rightarrow \frac{72x}{100} + 5600 = x$$

$$\Rightarrow 5600 = x - \frac{72x}{100}$$

$$\Rightarrow 5600 = \frac{100x - 72x}{100}$$

$$\Rightarrow \frac{28x}{100} = 5600$$

$$\Rightarrow x = \frac{5600 \times 100}{28} = 20000$$

$\therefore$ Anil's monthly salary = Rs. 20,000

Hence Anil's annual income = $12 \times 20000 = \text{Rs. } 2,40,000$.

- 3. In an education institution, there are 450 student, out of them 24% are girls. Fee paid by each girl is Rs. 520 per month and fee paid by each boy is 25% more than that of a girl. Calculate the total fee paid by all the girls and boys.**

Sol: Total no. of students = 450

No. of girls = 24% of 450

$$= \frac{24}{100} \times 450 = 108$$

No. of boys = $450 - 108 = 342$

Given that fee paid by each girl = 520

Fee paid by each boy = 25% more than girl's fee

$$= 520 + 25\% \text{ of } 520$$

$$= 520 + \frac{25}{100} \times 520$$

$$= 520 + 130 = 650$$

Total fee by all girls and boys

$$= \text{Fee paid by all girls} + \text{Fee paid by all boy}$$

$$= 108 \times 520 + 342 \times 650$$

$$= 278460$$

Type 3: Comparison of quantities on the same base value

- 1. In an examination, minimum passing percentage for girls is 35% and boys is 40%. A girl scored 238 marks and failed 42 marks. If a boy scored 298 marks, then how many marks he will required to pass?**

Sol: Let max. marks of the examination = x

$$\text{Marks scored by a girl} = 238$$

$$\text{Given that girl failed by 42 marks}$$

$$\Rightarrow \text{Minimum pass marks for girls} = 238 + 42$$

$$= 280$$

$$\text{Given that mini passing marks for girls} = 35\% \text{ of max. marks}$$

$$i.e., 280 = \frac{35}{100} \times x$$

$$\Rightarrow x = \frac{280 \times 100}{35} = 800$$

$\therefore$ Maximum marks of the examination = 800

Given that mini passing marks for boys = 40% of max. marks

$$= \frac{40}{100} \times 800 = 320$$

Marks scored by a boy = 298

$\therefore$ Marks required for the boy to pass = $320 - 298 = 22$

2. In an examination, it is required to get 65% of aggregate marks to pass. Sudheer gets 570 marks and he is failed by 8%. Find maximum aggregate marks.

Sol: Let aggregate marks = x

Given that required marks to pass = 65% of aggregate marks

$$= \frac{65}{100} \times x = \frac{65x}{100}$$

Sudheer's marks = 570

Given that sudheer failed by 8% of aggregate marks

i.e., $570 + 8\% \text{ of } x = \text{Passmarks}$

$$\Rightarrow 570 + \frac{8}{100} \times x = \frac{65x}{100}$$

$$\Rightarrow 570 = \frac{65x}{100} - \frac{8x}{100}$$

$$\Rightarrow 570 = \frac{57x}{100}$$

$$\Rightarrow x = \frac{570 \times 100}{57} = 1000$$

$\therefore$ Aggregate marks = 1000

- 3. In an entrance examination, Afreen scored 84% marks, Sadiya scored 92% marks and Chinni Scored 789 marks. The aggregate marks of the examination are 900. Find the average marks scored by all three girls.**

Sol: Given aggregate marks = 900

Afreen's marks = 84% of aggregate marks

$$= \frac{84}{100} \times 900 = 756$$

Sadiya's marks = 92% of aggregate marks

$$= \frac{92}{100} \times 900 = 828$$

Chinni's marks = 789

$$\begin{aligned}
 \therefore \text{Average marks} &= \frac{\text{Sum of marks scored by all girls}}{\text{No. of girls}} \\
 &= \frac{756 + 828 + 789}{3} \\
 &= \frac{2373}{3} = 791
 \end{aligned}$$

- 4. In an examination, Uday Sree scored 673 marks in an examination and Gayathri scored 82% of aggregate marks and Gayathri gets 65 marks more than Uday Sree. If minimum passing marks in the exam are 30% of aggregate marks, then how much more marks scored by Uday Sree than minimum passing marks.**

Sol: Let aggregate marks = x

Given UdaySree's marks = 673

$$\begin{aligned}
 \text{Gayathri's marks} &= 65 \text{ marks more than UdaySree's marks} \\
 &= 673 + 65 = 738
 \end{aligned}$$

Marks scored by Gayathri = 82% of aggregate marks

$$\begin{aligned}
 i.e., 738 &= \frac{82}{100} \times x \\
 \Rightarrow x &= \frac{738 \times 100}{82} = 900
 \end{aligned}$$

Aggregate marks = 900

Given that mini passing marks = 34% of max. marks

$$= \frac{30}{100} \times 900 = 270$$

Hence extra marks scored by Uday Sree than minimum passing marks = $673 - 270 = 403$.

Type 4: Increase or Decrease in Percentage

- 1. If the numerator of a fraction is decreased by 20% and the denominator is increased by 30%, the resultant fraction is $\frac{3}{5}$.**

Find the original fraction.

Sol:

Let the original fraction = $\frac{x}{y}$

Numerator decreased by 20% = $x - 20\%$ of x

$$\begin{aligned} &= x - \frac{20}{100} \times x \\ &= \frac{100x - 20x}{100} = \frac{80x}{100} \end{aligned}$$

Denominator increased by 30% = $y + 30\%$ of y

$$= y + \frac{20}{100} \times y$$

$$= \frac{100y + 20y}{100} = \frac{120y}{100}$$

$$\text{New Fraction} = \frac{\frac{80x}{100}}{\frac{120y}{100}}$$

$$= \frac{80x}{120y} = \frac{8x}{12y}$$

$$\text{Given that new fraction} = \frac{3}{5}$$

$$\Rightarrow \frac{8x}{12y} = \frac{3}{5}$$

$$\Rightarrow \frac{x}{y} = \frac{3}{5} \times \frac{12}{8} = \frac{9}{10}$$

$$\therefore \text{Original fraction} = \frac{9}{10}$$

- 2. Population of a city was 5,60,000 three years ago. If it increased by 5%, 4% and 10% respectively in the last three years, find the present population of that village.**

Sol: Three years ago the population of a city = 560000

First year population increased by 5%

$$= 560000 + 5\% \text{ of } 560000$$

$$= 560000 + \frac{5}{100} \times 560000$$

$$= 560000 \left(1 + \frac{5}{100} \right)$$

$$= 560000 \times \frac{105}{100} = 588000$$

Second year population increased by 4%

$$= 588000 + \frac{4}{100} \times 588000$$

$$= 588000 \left(1 + \frac{4}{100} \right)$$

$$= 588000 \times \frac{104}{100} = 611520$$

Thrid year population increased by 10%

$$= 611520 + \frac{10}{100} \times 611520$$

$$= 611520 \left(1 + \frac{10}{100} \right)$$

$$= 611520 \times \frac{110}{100} = 672672$$

$\therefore$ Present population of city = 672672

- 3. Height of Ramu increases at the rate of 20% every year. If the present height of Ramu is 540cm, what was the height of Ramu 2 years ago?**

Sol: Let 2 yers ago Ramu's height = x cm

After one year Ramu's height increased by 20%

$$\begin{aligned} &= x + \frac{20}{100} \times x \\ &= \frac{100x + 20x}{100} = \frac{120x}{100} = \frac{6x}{5} \end{aligned}$$

After 2 years (Presentyear) Ramu's height increased by 20%

$$\begin{aligned} &= \frac{6x}{5} + \frac{20}{100} \times \frac{6x}{5} \\ &= \frac{6x}{5} + \frac{6x}{25} = \frac{30x + 6x}{25} = \frac{36x}{25} \end{aligned}$$

Given that the present height of Ramu = 540 cm

$$\Rightarrow \frac{36x}{25} = 540$$

$$\Rightarrow x = \frac{540 \times 25}{36} = 375$$

$\therefore$ 2 yers ago Ramu's height = 375 *cm*

Type 5: New Effect with Increase or Decrease in Percentage

If the price of a commodity increases by $x\%$, then reduction in consumption as not to increase the expenditure is

$$\text{Reduction in consumption} = \left(\frac{x}{100 + x} \times 100 \right) \%$$

If the price of a commodity decreases by $x\%$, then the increase in consumption as not to decrease the expenditure is

$$\text{Increase in consumption} = \left(\frac{x}{100 - x} \times 100 \right) \%$$

- 1. The price of the oil is increased by 25%. If Swetha wants to keep her expenses on oil unchanged, then how much swetha will have to reduce the consumption?**

Sol: Given that price of the oil increased by 25%

Since Swetha wants to keep her expenses on oil remains unchanged

$$\begin{aligned}
 & \text{Reduction in consumption} \\
 &= \left(\frac{x}{100 + x} \times 100 \right) \% \\
 &= \frac{25}{100 + 25} \times 100 \\
 &= \frac{25}{125} \times 100 \\
 &= \frac{1}{5} \times 100 = 20\%
 \end{aligned}$$

- 2. The price of sugar is reduced by 4%. How many kgs of sugar can now be bought for the money which was sufficient to buy 6 kg of sugar earlier?**

Sol: Given that the price of sugar is reduced by 4%.

$$\begin{aligned}
 & \text{Increase in consumption} \\
 &= \left(\frac{x}{100 - x} \times 100 \right) \% \\
 &= \left(\frac{4}{100 - 4} \times 100 \right) \% \\
 &= \left(\frac{4}{96} \times 100 \right) \% = \frac{100}{24} = \frac{25}{6} \%
 \end{aligned}$$

$\therefore$ A person can now buy for the money which is sufficient to

buy 6 kg of sugar earlier

$$\begin{aligned} &= 6 + \frac{25}{6} \% \text{ of } 6 \\ &= 6 + \frac{\frac{25}{6}}{100} \times 6 \\ &= 6 + \frac{25}{100 \times 6} \times 6 \\ &= 6 + \frac{1}{4} \\ &= \frac{24 + 1}{4} = \frac{25}{4} = 6.25 \text{ kg} \end{aligned}$$

EXERCISE

1. Find 40% of 50% of 85% 500. **(Ans: 85)**
2. 20% of 350 – 40% of 140 = ?% of 250. **(Ans: 5.6 or $\frac{28}{5}$)**
3. Find the number whose $3\frac{1}{2}\%$ is 14. **(Ans: 400)**
4. Suraj spends 23% of his monthly income on education, 19% on travelling, 33% on food items, 16% on insurance policy. He saves Rs. 504. Calculate the total expenditure on education and food items. **(Ans: 3136)**
5. The seating capacity of bus 'A' is three times that of bus 'B'. The capacity of bus 'B' is 20% more than that of bus 'C'. If capacity of C is 2200 then find the capacity of bus A.

(Ans:7920)

6. In an examination, minimum passing percentage for girls is 40% and boys is 50%. A boy scored 483 marks and failed 67 marks. Find the minimum passing for girls. **(Ans:440)**
7. In an examination, A scored 25 marks less than B. B Scored 45 marks more than C. D Scored 75 marks which is 10 marks more than C. E Scored 50 marks less than maximum marks. If E got 45 marks more than A, then find the percentage of marks did E score in examination. **(Ans:70.4 %)**
8. Population of a country increases every year by 10%. If the population in 2021 was 3.4 crore, find the population in 2022 **(Ans:3.74 crore)**
9. Monthly salary of Swetha increases by 5% by every year. At present her salary is Rs. 50,000. What will be the salary after three years? **(Ans:Rs. 57,881.25)**



2.7 PROFIT AND LOSS

LEARNING OUTCOMES:

After completion of this topic, students will be able to

- Understand the terms like cost price, selling price, marked price, discount, profit, and loss.
- Solve problems on profit and loss and their practical applications in daily life.
- Learn about how to minimize loss and maximize profit margin.
- Understand the financial status of some particular things.

Introduction:

The terms profit and loss are used very often in our daily life. It helps us to have a better idea about our pockets. The concept of profit and loss is used to identify whether a transaction is profitable or not. When a person purchases a product for a certain price and sells it for a different price. During this transaction, he may get a profit or loss. There are various terms like cost price, selling price, marked price, discount, profit, and loss associated with the entire process of making a transaction. Let us learn about these terms.

- **Cost price (CP):** The price at which a product is purchased is called cost price.
- **Selling Price (SP):** The price at which a product is sold is called selling price.
- **Marked price (MP):** The price which is marked or listed on product to be sold is called marked price.
- **Discount:** Discount is a reduction in marked price which is usually expressed in percent.
- **Overhead Expenses:** If a product is purchased for some amount and there are some additional expenses like transportation, labour, commission etc., these are to be added to cost price such expenses are called overhead expenses.
- **Profit or Gain:** If the selling price of a product is more than cost price then the seller is said to have incurred a profit or gain.
- **Loss:** If the cost price of a product is more than selling price then the seller is said to have incurred a loss.

Basic Formulae Related to Profit and Loss:

1. Profit = Selling price – Cost price
2. Loss = Cost price – Selling price

$$3. \text{ Profit or Gain \%} = \frac{\text{Profit}}{\text{Cost Price}} \times 100 \%$$

$$4. \text{ Loss \%} = \frac{\text{Loss}}{\text{Cost Price}} \times 100 \%$$

$$5. \text{ SP} = \frac{100 + \text{Gain\%}}{100} \times \text{CP}$$

$$6. \text{ SP} = \frac{100 - \text{Loss \%}}{100} \times \text{CP}$$

$$7. \text{ CP} = \frac{100}{100 + \text{Gain\%}} \times \text{SP}$$

$$8. \text{ CP} = \frac{100}{100 - \text{Loss\%}} \times \text{SP}$$

WORKED OUT PROBLEMS

1.If Arun brought a bike for Rs. 54, 000 and sold for Rs. 63420, find the profit percent.

Sol: Cost Price CP = 54000

Selling Price SP = 63420

$$\text{Profit} = \text{SP} - \text{CP}$$

$$= 63420 - 5400$$

$$= 9420$$

$$\begin{aligned}
 \text{Profit}\% &= \frac{\text{Profit}}{\text{CP}} \times 100\% \\
 &= \frac{9420}{54000} \times 100 \\
 &= \frac{157}{9} \\
 &= 17.44\% \text{ Or } 17\frac{4}{9}\%
 \end{aligned}$$

2.If cost price is equal to 300% of selling price, then find profit or loss percent.

Sol: Let selling price $\text{SP} = x$

Given that $\text{CP} = 300\% \text{ of } x$

$$\begin{aligned}
 &= \frac{300}{100} \times x \\
 &= 3x
 \end{aligned}$$

Since $\text{CP} > \text{SP}$

$$\begin{aligned}
 \text{Loss} &= \text{CP} - \text{SP} \\
 &= 3x - x = 2x
 \end{aligned}$$

$$\therefore \text{Loss}\% = \frac{\text{Loss}}{\text{CP}} \times 100\%$$

$$= \frac{2x}{3x} \times 100$$

$$= \frac{200}{3}$$

$$= 66\frac{2}{3}\%$$

3. The ratio of cost price and selling price is 7: 9. What is gain percent?

Sol: Given that the ratio of CP and SP is 7: 9

$$\text{Let CP} = 7x$$

$$\text{SP} = 9x$$

$$\text{Gain} = \text{SP} - \text{CP}$$

$$= 9x - 7x$$

$$= 2x$$

$$\text{Gain}\% = \frac{\text{Gain}}{\text{CP}} \times 100\%$$

$$= \frac{2x}{7x} \times 100 = \frac{200}{7}$$

$$= 28\frac{4}{7}\%$$

4. If cost price of 20 apples is equal to selling price of 12 apples,

then calculate gain percent.

Sol: Let CP of 1 apple = x

$$\Rightarrow \text{CP of 20 apples} = 20x$$

$$\Rightarrow \text{CP of 12 apples} = 12x$$

$$\begin{aligned}\text{SP of 12 apples} &= \text{CP of 20 apples} \\ &= 20x\end{aligned}$$

$$\begin{aligned}\text{Gain} &= \text{SP of 12 apples} - \text{CP of 12 apples} \\ &= 20x - 12x = 8x\end{aligned}$$

$$\begin{aligned}\text{Gain\%} &= \frac{\text{Gain}}{\text{CP of 12 apples}} \times 100\% \\ &= \frac{8x}{12x} \times 100 = \frac{2}{3} \times 100 = 66\frac{2}{3}\%\end{aligned}$$

5. A seller sells 345 books and he get profit of selling price of 50 books. Calculate his profit percent.

Sol:

$$\text{Profit gained by seller} = \text{SP of 50 books}$$

$$\begin{aligned}\Rightarrow \text{CP of 345 books} &= \text{SP of } (345 - 50) \text{ books} \\ &= \text{SP of 295 books}\end{aligned}$$

Let CP of one book = x

$\Rightarrow$ CP of 345 books = $345x$

$\Rightarrow$ CP of 295 books = $295x$

$\Rightarrow$ SP of 295 books = CP of 345 books = $345x$

Profit = SP of 295 books – CP of 295 books

$$= 345x - 295x = 50x$$

$$\text{Profit \%} = \frac{\text{Profit}}{\text{CP of 295 books}} \times 100\%$$

$$= \frac{50x}{295x} \times 100$$

$$= \frac{1000}{59} = 16.9\%$$

6. Lucky buys a rice cooker for Rs. 2400 and sells it for profit of 30%. Find the selling price of rice cooker.

Sol: Given CP of rice cooker = 2400

$$\text{Profit\%} = 30\%$$

$$\text{SP of rice cooker} = \frac{100 + \text{profit \%}}{100} \times \text{CP}$$

$$= \frac{100 + 30}{100} \times 2400$$

$$= 130 \times 24 = 3120$$

7. Sujatha buys two gold rings each cost is 67,000. She sells one ring with 17% of profit and other one with 9% of loss. Calculate her net profit or loss percent.

Sol: Each gold ring cost = 67000

$$\text{1st ring profit \%} = 17\%$$

$$\text{SP of 1st ring is} = \frac{100 + \text{profit \%}}{100} \times \text{CP}$$

$$= \frac{100 + 17}{100} \times 67000$$

$$= 117 \times 670$$

$$= 78390$$

$$\text{2nd ring loss \%} = 9\%$$

$$\text{SP of 2nd ring is} = \frac{100 - \text{loss\%}}{100} \times \text{CP}$$

$$= \frac{100 - 9}{100} \times 67000$$

$$= 91 \times 670$$

$$= 60970$$

$$\text{Total cost of two gold rings} = 67000 + 67000 = 134000$$

$$\text{Total SP of two gold rings} = 78390 + 60970 = 139360$$

$$\text{Net profit} = \text{SP} - \text{CP} = 139360 - 134000 = 5360$$

$$\begin{aligned}\therefore \text{Net profit \%} &= \frac{\text{Net Profit}}{\text{CP}} \times 100\% \\ &= \frac{5360}{134000} \times 100 = 4\%\end{aligned}$$

8. Aadi buys two desktops, he sells first desktop with loss of 20% and second desktop with gain of 25%. If he sells each desktop for Rs. 23,000 then find the cost price of both tables.

Sol: Given that SP of each desktop = Rs. 23,000

$$\text{First desktop's loss percent} = 20\%$$

Then, CP of first desktop

$$\begin{aligned}&= \frac{100}{100 - \text{loss}\%} \times \text{SP} \\ &= \frac{100}{100 - 20} \times 28000 \\ &= \frac{2800000}{80} \\ &= 35000\end{aligned}$$

$$\text{Second desktop's gain percent} = 25\%$$

$$\begin{aligned}
 \text{Then CP of second desktop} &= \frac{100}{100 + \text{gain}\%} \times \text{SP} \\
 &= \frac{100}{100 + 25} \times 28000 \\
 &= \frac{2800000}{125} \\
 &= 22400
 \end{aligned}$$

$\therefore$ CP of both desktops

$$\begin{aligned}
 &= \text{CP of 1st desktop} + \text{CP of 2nd desktop} \\
 &= 35000 + 22400 \\
 &= \text{Rs. } 57400
 \end{aligned}$$

9. Neha sells a toy for Rs. 546 at loss of 9%. How much should Neha sell it to get 6% of profit?

Sol: Given that SP of toy = Rs. 546

$$\text{Loss \%} = 9\%$$

$$\begin{aligned}
 \text{Then Cost Price of toy} &= \frac{100}{100 - \text{loss}\%} \times \text{SP} \\
 &= \frac{100}{100 - 9} \times 546 \\
 &= \frac{54600}{91} = 600
 \end{aligned}$$

Now, CP = Rs. 600

If Neha wants to get 6% of profit,

$$\begin{aligned}\text{Then desired SP} &= \frac{100 + \text{profit}\%}{100} \times \text{CP} \\ &= \frac{100 + 6}{100} \times 600 \\ &= 106 \times 6 = 636\end{aligned}$$

If Neha sells the toy for Rs. 636, then she will get 6% of profit.

- 10. Rajkumar sells a watch to his friend by 20% of loss. If his friend sells that watch for Rs. 3500 with 25% of profit the find cost price of watch for Rajkumar.**

Sol: SP of watch by friend = Rs. 3500

Profit % of friend = 25%

$$\begin{aligned}\text{Then CP of watch for friend} &= \frac{100}{100 + \text{Profit}\%} \times \text{SP} \\ &= \frac{100}{100 + 25} \times 3500 \\ &= \frac{350000}{125} = 2800\end{aligned}$$

Since SP of watch by Rajkumar = CP of watch for friend

$$= \text{Rs. } 2800$$

$$\text{Loss \% of Rajkumar} = 20\%$$

$$\begin{aligned}\therefore \text{CP of watch for Rajkumar} &= \frac{100}{100 - \text{loss}\%} \times \text{SP} \\ &= \frac{100}{100 - 20} \times 3500 \\ &= \frac{350000}{80} \\ &= \text{Rs. } 4375\end{aligned}$$

11. Sarojini buys a watch and sells it by 8% loss. If she sells the watch for Rs. 200 more, she will get 8% profit. For what price sarojini sells the watch?

Sol: Let SP of the watch = x

$$\text{Given that Loss \%} = 8\%$$

$$\begin{aligned}\text{Then the CP of watch} &= \frac{100}{100 - \text{loss \%}} \times \text{SP} \\ &= \frac{100}{100 - 8} \times x = \frac{100x}{92}\end{aligned}$$

If Sorojini wants to get 8% of profit,

$$\text{then SP} = \frac{100 + \text{profit}\%}{100} \times \text{CP}$$

$$= \frac{100 + 8}{100} \times \frac{100x}{92}$$

$$= \frac{108x}{92} = \frac{27x}{23}$$

$$\therefore \text{SP with profit} - \text{SP with loss} = 200$$

$$\Rightarrow \frac{27x}{23} - x = 200$$

$$\Rightarrow \frac{27x - 23x}{23} = 200$$

$$\Rightarrow \frac{4x}{23} = 200$$

$$\Rightarrow x = \frac{200 \times 23}{4} = 1150$$

Hence the selling price of the watch = 1150.

12. Swetha buys sugar for Rs. 540. Due to reduction of 20% in the price of the sugar, she gets extra 5 Kg of sugar. Find the original cost of sugar per one Kg.

Sol: Let original price of sugar per 1 Kg = x

Given that reduction of price = 20% of original price

$\Rightarrow$ New price of sugar per 1 Kg = $x - 20\% \text{ of } x$

$= 80\% \text{ of } x$

$$= \frac{80}{100} \times x = \frac{4x}{5}$$

$$\text{No. of Kgs of suger for Rs. 540 as per original rate} = \frac{540}{x}$$

$$\text{No. of Kgs of suger for Rs. 540 as per new rate} = \frac{540}{\frac{4x}{5}}$$

$$= \frac{540 \times 5}{4x} = \frac{675}{x}$$

$$\text{No. of Kgs as per new rate} - \text{No. of Kgs as per Original rate} = 5$$

$$\Rightarrow \frac{675}{x} - \frac{540}{x} = 5$$

$$\Rightarrow \frac{675 - 540}{x} = 5$$

$$\Rightarrow \frac{135}{x} = 5$$

$$\Rightarrow x = \frac{135}{5} = 27$$

$\therefore$ Original price of sugar per 1 Kg = Rs. 27

1. Geetha sells a bag with 10% discount on marked price. If marked price of the bag is 20% more than cost price then find profit percent.

Sol: Let the cost price of item = x

Then, marked price = 20% more than CP

$$= x + 20\% \text{ of } x$$

$$= 120\% \text{ of } x$$

$$= \frac{120x}{100} = \frac{6x}{5}$$

Given that, discount = 10% of MP

$$= \frac{10}{100} \times \frac{6x}{5} = \frac{6x}{50}$$

We have, SP = MP – Discount

$$= \frac{6x}{5} - \frac{6x}{50}$$

$$= \frac{60x - 6x}{50} = \frac{54x}{50}$$

Profit = SP – CP

$$= \frac{54x}{50} - x$$

$$= \frac{54x - 50x}{50}$$

$$= \frac{4x}{50} = \frac{2x}{25}$$

$$\text{Profit}\% = \frac{\text{Gain}}{CP} \times 100$$

$$= \frac{\frac{2x}{25}}{x} \times 100$$

$$= \frac{2}{25} \times 100 = 8\%$$

- 2. How much percent above the cost price should a seller marks his goods such that he gains 20% after allowing 20% of discount on marked price.**

Sol: Suppose the cost price = Rs. 100

$$\text{Gain}\% = 20\%$$

$$\therefore SP = \frac{100 + \text{Gain}\%}{100} \times CP$$

$$= \frac{100 + 20}{100} \times 100$$

$$= 120$$

Let Marked price = x

$$\text{Discount} = 20\% \text{ of } x = \frac{20x}{100} = \frac{x}{5}$$

We have, $SP = MP - \text{Discount}$

$$\Rightarrow 120 = x - \frac{x}{5}$$

$$\Rightarrow 120 = \frac{5x - x}{5}$$

$$\Rightarrow 120 \times 5 = 4x$$

$$\Rightarrow x = \frac{600}{4} = 150$$

$\therefore$ Marked price = Rs. 150

Hence the marked price is 50% above the cost price

- 3. Find the single discount which is equivalent to the three successive discounts of 20%, 15% and 10%.**

Sol:

Single discount for two successive discounts x and y

$$= x + y - \frac{xy}{100}$$

Single discount for two successive discounts 20% and 15%

$$= 20 + 15 - \frac{20 \times 15}{100}$$

$$= 35 - 3 = 33\%$$

Single discount for two successive discounts 33% and 10%

$$= 33 + 10 - \frac{33 \times 10}{100}$$

$$= 43 - 3.3 = 40.3\%$$

∴ Single discount for three successive discounts 20%, 15% and 10%

$$= 40.3\%$$

EXERCISE

1. The difference between selling price and cost price of a book is Rs. 250, if the profit percent is 20%, then find the cost price of the book. **(Ans:Rs.1250)**
2. The ratio of CP and SP is 8: 14. Find the profit percent. **(Ans: 75%)**
3. Amit sells 20 toys and he loses an amount equal to selling price of 5 toys. Find his loss percent. **(Ans:20%)**
4. Surya buys a cube for Rs.100 and sells it at 4% of loss. Find the selling price of the cube. **(Ans: Rs.96)**
5. Roopa sells her earphones for Rs. 371, and gets profit of 6%. How much should Roopa sells her earphones to get 8% of profit. **(Ans: Rs.378)**

6. Afroz buys two make-up kits each for Rs. 750. She sells a make-up kit for 6% of gain and other for 4% of loss. Find her net gain or loss percent. **(Ans: 1%)**
7. Rahul buys a painting and saved Rs. 900 by getting a discount of 20%. Find the amount paid by the Rahul. **(Ans: Rs. 3,600)**
- (Hint: Amount paid by Rahul = marked price - savings).
8. Varun marked up his goods by 20% and allows 20% of discount n marked price . Find his profit or loss percent. **(Ans: Loss%=4%)**



2.8. PARTNERSHIP

LEARNING OUTCOMES

After reading this topic a student will be able to -

- ❖ Understand the terms partnership, partners, types of partnership, types of partners.
- ❖ Solve problems on partnership

Introduction:

Partnership is a deal where two or more persons (known as partners) make an association and invest money (known as capital) for running a certain business. After specific time period the business may generate profit or loss. This profit or loss is distributed among the partners in the ratio of their invested money and time period of investment.

Types of Partnership:

Partnership is mainly of two types namely simple partnership and compound partnership.

1. Simple Partnership: A partnership in which the partners invest same capital for different time periods or different capitals for same time period is called as simple partnership.

In this partnership the profit or loss is distributed among the partners in the ratio of their investment or time period of investment.

2. Compound Partnership: A partnership in which the partners invest different capitals for different time periods is called as compound partnership.

In this partnership the profit or loss not only depends on their investments but also on the time period of their investments.

Types of Partners:

There are two types of partners namely 1. Active or working partner and 2. Sleeping partner.

1. Active or working partner: A partner who not only invests money but also involves in the business activities is known as active partner.

2. Sleeping Partner: A partner who only invests money and does not involve in business activities is known as sleeping partner.

Problem solving techniques:

Case I - In case of Simple Partnership:

❖ When two partners A and B invest the same amount for different time periods, then profit or loss is distributed among them in the ratio of their time periods.

i.e. Ratio of profit or loss = Ratio of their time periods

❖ When two partners A and B invest different amounts for same time periods, then profit or loss is distributed among them in the ratio of their investments.

i.e. Ratio of profit or loss = Ratio of their investments

WORKED OUT PROBLEMS

1: Raman and Gopal start a business by investing ₹ 5000 and ₹ 9000, respectively. Find the ratio of their profits after 1 year.

Sol. Here the time period is same.

Then we know that,

Ratio of their profits = Ratio of their investments

$$= ₹ 5000 : ₹ 9000 = \frac{5000}{9000} = \frac{5}{9} = 5 : 9$$

Hence required ratio = 5 : 9

2: A and B jointly start a business. A's investment is equal to four times B's investment. Find A's share in the annual profit of ₹ 78,000.

Sol. Let B's investment = ₹ x

Then A's investment = ₹ $4x$

We know that, Ratio of their profits = Ratio of their investments

$$\text{Hence, ratio of their profits} = 4x : x = 4 : 1 = \frac{4}{1}$$

Given, their annual profit = ₹ 78,000

$$\text{Now A's share} = \frac{4}{4+1} \times 78,000 = \frac{4}{5} \times 78,000 = ₹ 62,400.$$

3: A, B and C started a business by investing ₹ 90,000, ₹ 1,20,000 and ₹ 1,50,000 respectively. Find the profit of each out of an annual profit of ₹ 76,800.

Sol. Here the time period is same.

$$\begin{aligned} \therefore \text{Ratio of their profits} &= \text{Ratio of their investments} \\ &= 90,000 : 1,20,000 : 1,50,000 \end{aligned}$$

$$= 3 : 4 : 5$$

$$\text{A's profit} = \frac{3}{3+4+5} \times 76800 = \frac{3}{12} \times 76800 = ₹ 19,200$$

$$\text{B's profit} = \frac{4}{3+4+5} \times 76800 = \frac{4}{12} \times 76800 = ₹ 25,600$$

$$\text{C's profit} = \frac{5}{3+4+5} \times 76800 = \frac{5}{12} \times 76800 = ₹ 32,000$$

4: Gopi and Amar invested ₹ 16,000 and ₹ 10,000 for a period of 3 years. After

3 years they earned a profit of ₹ 65,650. What will be the shares of Gopi and Amar out of this profit?

Sol. Here the time period is same.

∴ Ratio of their profits = Ratio of their investments

$$= 16,000 : 10,000$$

$$= 8 : 5$$

$$\text{Gopi's profit} = \frac{8}{8+5} \times 65650 = \frac{8}{13} \times 65650 = ₹ 40,400$$

$$\text{Amir's profit} = \frac{5}{8+5} \times 65650 = \frac{5}{13} \times 65650 = ₹ 25,250$$

5: A, B and C start a business with investing ₹ 15,000 each. A remains in partnership for 9 months, B for 6 months and C for 12 months. Then find the ratio of their profits.

Sol. Here the capital amount is same.

∴ Ratio of their profits = Ratio of their time period of investment

$$\Rightarrow \text{A's profit} : \text{B's profit} : \text{C's profit} = 9 : 6 : 12 = 3 : 2 : 4$$

6: A, B and C starts a business where A invests 2 times as much as B invests and B invests one- third of what C invests. At the end of the year the profit earned is ₹ 7200. What are the shares of A, B and C?

Sol. Let C's investment = ₹ x .

$$\text{Then B's investment} = ₹ \frac{1}{3} \times x = ₹ \frac{x}{3}$$

$$\text{A's investment} = ₹ \frac{x}{3} \times 2 = ₹ \frac{2x}{3}$$

Here the time period is same.

∴ Ratio of their profits = Ratio of their investments

$$\Rightarrow \text{Ratio of their profits} = \frac{2x}{3} : \frac{x}{3} : x = 2 : 1 : 3.$$

$$\text{Now A's share} = \frac{2}{2+1+3} \times 7200 = \frac{2}{6} \times 7200 = ₹ 2,400$$

$$\text{B's share} = \frac{1}{2+1+3} \times 7200 = \frac{1}{6} \times 7200 = ₹ 1,200$$

$$\text{C's share} = \frac{3}{2+1+3} \times 7200 = \frac{3}{6} \times 7200 = ₹ 3,600$$

Case II - In case of Compound Partnership

When two partners A and B invest amount ₹ x for a period of p months and ₹ y for a period of q months respectively, then profit or loss is distributed among them in the ratio of the product of their amount of capital invested and time period of investment.

i.e. Ratio of their profit or loss = Ratio of their (capital invested $\times$ time period of investment)

$$\text{i.e. Profit of A : Profit of B} = xp : yq$$

WORKED OUT PROBLEMS:

1: Suman starts a business with ₹ 5,000 and Raman joins him after 4 months with ₹9,000. Find the ratio of their profits at the end of the year.

Sol. Given, Suman's investment = ₹ 5,000

Suman's time period of investment = 1 yr = 12 months

Raman invested after 4 months.

i.e. Raman's time period of investment = $(12 - 4)$ months
= 8 months.

Raman's investment = ₹ 9,000

We know that, ratio of profits = ratio of their (capital invested $\times$ time period of investment)

$\Rightarrow$ Suman's share : Raman's share = $5000 \times 12 : 9000 \times 8$

$$= 60 : 72 = 5 : 6$$

2: A starts a business with ₹ 3,000 and B joins the business 4 months later with an investment of ₹ 6,000. After 1 year they earned a profit of ₹ 35,000. Find the share of A and B.

Sol. A 's capital = ₹ 3,000

A 's time period of investment = 1 yr = 12 months

B 's capital = ₹ 6,000

B 's time period of investment = $(12 - 4)$ months = 8 months.

We know that,

ratio of profits = ratio of their (capital invested $\times$ time period of investment)

$$\Rightarrow A \text{'s share} : B \text{'s share} = 3000 \times 12 : 6000 \times 8 = 3 : 4$$

$$A \text{'s share} = \frac{3}{3+4} \times 35,000 = \frac{3}{7} \times 35,000 = ₹ 15,000$$

$$B \text{'s share} = \frac{4}{3+4} \times 35,000 = \frac{4}{7} \times 35,000 = ₹ 20,000$$

Problem solving tricks and important formulae

- ❖ If $x_1 : x_2 : x_3 : \dots$ is the ratio of investments and $P_1 : P_2 : P_3 : \dots$ is the ratio of profits, then the ratio of time periods of investment is given by

$$\frac{P_1}{x_1} : \frac{P_2}{x_2} : \frac{P_3}{x_3} : \dots$$

- ❖ If $P_1 : P_2 : P_3 : \dots$ is the ratio of profits and $t_1 : t_2 : t_3 : \dots$ is the ratio of time periods then ratio of their investments is given by

$$\frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} : \dots$$

WORKED OUT PROBLEMS

1: A, B and C invested capitals in the ratio of 5 : 2 : 3. At the end of the business, they received the profit in the ratio of 5 : 3 : 6. Find the ratio of time periods for which they contributed their capitals.

Sol. Given, ratio of investments = $x_1 : x_2 : x_3 = 5 : 2 : 3$.

$$\text{Ratio of profits} = P_1 : P_2 : P_3 = 5 : 3 : 6$$

$$\text{Let the ratio of time periods} = t_1 : t_2 : t_3$$

We know that,

Ratio of profits = Ratio of (capital invested $\times$ time period of investment)

$$\Rightarrow 5 : 3 : 6 = 5t_1 : 2t_2 : 3t_3$$

Taking the first two terms of the ratio, we get

$$\frac{5}{3} = \frac{5t_1}{2t_2} \Rightarrow \frac{t_1}{t_2} = \frac{2}{3} \Rightarrow t_1 : t_2 = 2 : 3 \quad \text{-----} \quad (1)$$

Now taking the last two terms of the ratio, we get

$$\frac{3}{6} = \frac{2t_2}{3t_3} \Rightarrow \frac{t_2}{t_3} = \frac{3}{4} \Rightarrow t_2 : t_3 = 3 : 4 \quad \text{-----} \quad (2)$$

From equations (1) and (2), we get

$$t_1 : t_2 : t_3 = 2 : 3 : 4$$

Hence ratio of their time periods = 2 : 3 : 4

Alternate method:

Given, $x_1 : x_2 : x_3 = 5 : 2 : 3$.

$$P_1 : P_2 : P_3 = 5 : 3 : 6$$

$$\begin{aligned} \text{Required ratio} &= \frac{P_1}{x_1} : \frac{P_2}{x_2} : \frac{P_3}{x_3} = \frac{5}{5} : \frac{3}{2} : \frac{6}{3} = 1 : \frac{3}{2} : \frac{2}{1} \\ &= 1 \times 2 : \frac{3}{2} \times 2 : \frac{2}{1} \times 2 = \\ &2 : 3 : 4 \end{aligned}$$

2: A, B and C invested certain amounts for time periods in the ratio of 5 : 6 : 8 at the end of the business, they received the profit in the ratio of 5 : 3 : 12. Find the ratio of their investments.

Sol. Given, ratio of time periods = $t_1 : t_2 : t_3 = 5 : 6 : 8$

$$\text{Ratio of profits} = P_1 : P_2 : P_3 = 5 : 3 : 12$$

Let the ratio of capitals = $x_1 : x_2 : x_3$

We know that,

Ratio of profits = Ratio of (capital invested $\times$ time period of investment)

$$\Rightarrow 5 : 3 : 12 = 5x_1 : 6x_2 : 8x_3$$

Taking the first two terms of the ratio, we get

$$\frac{5}{3} = \frac{5x_1}{6x_2} \Rightarrow \frac{x_1}{x_2} = \frac{2}{1} \Rightarrow x_1 : x_2 = 2 : 1 \quad \text{-----}$$

(1)

Now taking the last two terms of the ratio, we get

$$\frac{3}{12} = \frac{6x_2}{8x_3} \Rightarrow \frac{x_2}{x_3} = \frac{1}{3} \Rightarrow x_2 : x_3 = 1 : 3 \quad \text{-----}$$

(2)

From equations (1) and (2), we get

$$x_1 : x_2 : x_3 = 2 : 1 : 3$$

Hence ratio of their capitals = 2 : 1 : 3

Alternate method:

Given, $t_1 : t_2 : t_3 = 5 : 6 : 8$

$$P_1 : P_2 : P_3 = 5 : 3 : 12$$

$$\text{Required ratio} = \frac{P_1}{t_1} : \frac{P_2}{t_2} : \frac{P_3}{t_3} = \frac{5}{5} : \frac{3}{6} : \frac{12}{8} = 1 : \frac{1}{2} : \frac{3}{2}$$

$$= 2 : 1 : 3$$

Exercise

- 1) *A* and *B* start a business by investing ₹ 4000 and ₹ 7000, respectively. Find the ratio of their profits after 1 year.
- 2) *A* and *B* jointly start a business. *A*'s investment is equal to three times *B*'s investment. Find *A*'s share in the annual profit of ₹ 52,000.
- 3) *A*, *B* and *C* enter into a business. *A* invests three times as much as *B* invests and *B* invests two-thirds of what *C* invests. At the end of the year, they earned a profit of ₹ 6,600. What is the share of *B*?
- 4) *A*, *B* and *C* started a business by investing ₹ 12,000, ₹ 1,35,000 and ₹ 1,50,000 respectively. Find the share of *C*, out of annual profit of ₹ 56,700.
- 5) *A*, and *B* started a business by investing ₹ 8,000, ₹ 7,000 and respectively. What is the share of *A*, out of annual profit of ₹ 22,500?
- 6) In a partnership business, Vishal invested ₹ 4,200 and Kushal invested a certain amount. Out of the overall profit of ₹ 600, *A*'s share is ₹ 320. Then find the amount invested by *B*.
- 7) *A* starts a business with ₹ 4,000 and *B* joins the business 4 months later with an investment of ₹ 5,000.

After 1 year they earned a profit of ₹ 22,000. Find the share of A and B.

8) A starts a business with ₹ 2,000 and B joins him after 3 months with ₹8,000. Find the ratio of their profits at the end of the year.

9) A, B and C invested capitals in the ratio of 2 : 3 : 5. At the end of the business, they received the profit in the ratio of 5 : 3 : 12. Find the ratio of time periods for which they contributed their capitals.

Answers

1. 4 : 7

2. ₹ 39,000

3. ₹ 1,200

4. ₹ 21,000

5. ₹ 12,000

6. ₹ 3,675

7. ₹ 12,000 and

₹ 10,000 respectively

8. ₹1 : 3

9. 25 : 10 : 24

Web resources for advanced learners:

- ❖ <https://byjusexamprep.com/how-to-solve-partnership-problems-i>
- ❖ https://youtu.be/hn9TKnr8L_8
- ❖ <https://youtu.be/iSFWq6Aj2X0>
- ❖ <https://youtu.be/vk5RyHTX8wA>



2.9. SIMPLE INTEREST

Learning Outcomes:

After reading this a student will learn to:

- Understand the terms principal , interest, Simple interest .
- Understand the concept of simple interest
- Solve problems involving investments and the simple interest formula.
- Understand logic and solve the problems
- Understand enhance the problem solving skills

Introduction:

If a person borrows some money from some one ,then while making a payment to the lender, the borrower pay some extra money. This extra money is called the interest .The money Borrowed is called principal. The total payment of the principal along with the interest is called the Amount. The number of years/months/days for which the money has been kept by the borrower is called time. The extra money paid on Rs 100per year is called the rate percent per annum.

The interest is of two types.(1) simple interest (S.I) (2) Compound interest(C.I)

Principal(P):

The Money borrowed or lent out for a certain period is called the principal or the sum.

Interest :

Extra money paid for using other's money is called interest.

Simple Interest(S.I.):

If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called Simple interest.

Time: Time is duration for which the principal amount is given to some one.

Amount(A): When a person takes an amount from some one,he/she has to return the principal borrowed plus the interest amount, and this total amount is called Amount.

2.9.1.3 Notations:

Principal =P , Rate = R% per annum(p.a), Time=T years ;
A=Amount

Simple Interest = S.I.

2.9.1.4 Formulae:

$$\text{i. } S.I. = \frac{P \times R \times T}{100}$$

$$\text{ii. } P = \frac{100 \times S.I.}{R \times T}$$

$$\text{iii. } R = \frac{100 \times S.I.}{P \times T}$$

$$\text{iv. } T = \frac{100 \times S.I.}{P \times R}$$

v. Amount = principal + Simple interest

$$A = P + S.I.$$

$$A = P + PRT$$

$$= P(1 + RT)$$

If borrowed money is paid through equal instalments:

Let borrowed amount be Rs .P

amount of equal instalments =Rs x

Number of instalments = n

Rate percent per annum = r% per annum

$$\text{Then, } P = n \times \left[1 + \frac{r(n-1)}{200} \right]$$

WORKED OUT PROBLEMS

1: Find the simple interest on Rs.200 for year at 6% per annum.

Ans: Here $P = \text{Rs } 200$, $T = 2$ years, $R = 6\%$

$$SI = \frac{P \times R \times T}{100} = \frac{200 \times 6 \times 2}{100} = \text{Rs.} 48$$

2: In what time, Rs 1200 will become Rs 1450 when annual rate of interest is 20% ?

Ans: Here $P = \text{Rs } 1200$, $A = \text{Rs } 1450$, $R = 20\%$

$$A = P + S.I.$$

$$S.I. = A - P = 1450 - 1200 = \text{Rs } 250$$

$$T = \frac{100 \times S.I.}{P \times R} = \frac{100 \times 250}{1200 \times 20} = \frac{25}{24} = 1\frac{1}{24} \text{ years}$$

3: Mahesh borrowed Rs. 3,000 from his friend Suresh at 15% per annum for 3 years. Find interest and money returned by Mahesh to Suresh.

Ans: Here $P = \text{Rs.} 3,000$

$R = 15\%$ per annum,

$T = 3$ years

$$\therefore S.I. = \frac{P \times R \times T}{100} = \frac{3000 \times 15 \times 3}{100} = \text{Rs.} 1,350$$

$$\therefore A = P + S.I. = 3000 + 1,350 = \text{Rs.} 4,350$$

4. The simple interest accrued on an amount at the end of 5 years at 12.5% is Rs 1575.What is the amount ?

Ans: Here T= 5 years , S.I.=Rs 1575, R=12.5%

$$\therefore P = \frac{SI \times 100}{T \times R} = \frac{1575 \times 100}{5 \times 12.5} = \text{Rs.}2520$$

5. The principal interest accrued on a sum of certain principal is Rs.1200 in 4 years at the rate of 8% per annum. What would be the simple interest accrued on thrice of that principal at the rate of 6%per annum in 3 years ?

Ans: Here ,S.I. = Rs 1200, T=4 years, R=8%

$$\therefore P = \frac{SI \times 100}{T \times R} = \frac{1200 \times 100}{4 \times 8} = \text{Rs.}3750$$

Again , $P = 3 \times 3750 = \text{Rs.}11250$

T = 3 years , R = 6%

$$S.I. = \frac{P \times R \times T}{100} = \frac{11250 \times 3 \times 6}{100} = \text{Rs.}2025$$

6. A sum of Rs.5000 amounts to Rs.6050 in 2 years, what is the rate of interest ?

Ans: P = Rs.5000, A = Rs.6050 , T=2 years ,R = ?

S.I.=6050-5000 = Rs.1050

$$R = \frac{S.I. \times 100}{P \times T} = \frac{1050 \times 100}{5000 \times 2} = 10.5\%$$

7. How much time will it take for an amount of Rs 900 to yield Rs 81 as interest at 4.5% per annum of simple interest ?

Ans: Here $P = \text{Rs } 900$, $S.I. = \text{Rs } 81$, $R = 4.5\%$

$$T = \frac{100 \times S.I.}{P \times R} = \frac{100 \times 81}{900 \times 4.5} = 0.2\%$$

8. Srinivasan invests two equal amounts in two banks giving 10% and 12% rate of interest, respectively. At the end of the year the interest earned is Rs 1650. Find the sum invested in each ?

Ans: Here $T = 1$ year, Interest rates $R_1 = 10\%$, $R_2 = 12\%$

Let the sum invested be Rs x

Then,

$$\frac{X \times R_1 \times T}{100} + \frac{X \times R_2 \times T}{100} = 1650$$

$$\frac{X \times 10 \times 1}{100} + \frac{X \times 12 \times 1}{100} = 1650$$

$$\frac{10X}{100} + \frac{12X}{100} = 1650$$

$$22X = 1650 \times 100$$

$$X = \frac{1650 \times 100}{22}$$

$$X = \text{Rs. } 7500$$

9. Rs 800 becomes Rs. 956 in 3 years at certain simple rate of interest. If the rate of interest is increased by 4%, what amount will Rs 800 become in three years ?

Ans: Here

$$P = \text{Rs } 800, A = \text{Rs } 956$$

$$S.I. = 956 - 800 = \text{Rs.}156, T = 3 \text{ years}$$

$$R = \frac{S.I. \times 100}{P \times T} = \frac{156 \times 100}{800 \times 3} = 6.5\%$$

$$\text{Now new rate} = 6.5 + 4 = 10.5\%$$

$$S.I. = \frac{P \times R \times T}{100} = \frac{800 \times 10.5 \times 3}{100} = \text{Rs.} 252$$

$$\text{Amount}(A) = P + S.I. = 800 + 252 = \text{Rs.} 1052$$

10. What annual instalment will discharge a debt of Rs.4200 due in 5 years at 10% simple interest ?

Ans: Here , $P = \text{Rs.} 4200$

$$n=5, r = 10\% \quad x = \text{annual instalment} = ?$$

$$P = nx \times \left(1 + \frac{r(n-1)}{200} \right)$$

$$4200 = 5x \times \left(1 + \frac{10(5-1)}{200} \right)$$

$$4200 = 5x \times \left(\frac{240}{200} \right)$$

$$x = \text{Rs.} 700 \text{ per year.}$$

EXERCISE

1. Find the simple interest on Rs. 68000 at $16\frac{2}{3}\%$ for annum for 9 months.

2. A sum of money amounts to Rs.5200 in 5 years and to Rs.5680 in 7 years at simple interest. The Rate of interest per annum is?

3.Siva borrow Rs.520 from Arun a simple interest of 13%per annum. What amount of money should siva pay to Arun after 6 months to be absolved of the debt ?

4If Rs.4 becomes Rs.10 in 50 years at simple interest , the rate % per annum is ?

5. A sum fetched a total simple interest of Rs. 929.20 at the rate of 8% per annum in 5 years.what is the sum ?

Answers:

1.Rs.8500 2. 6% 3. Rs.553.80 4. 3% 5. Rs. 2323



2.10 COMPOUND INTEREST

LEARNING OUT COMES:

After reading this a student will learn to:

- Understand and Identify types interest problems
- Understand and logic and solve the problems
- Understand enhance the problem solving skills
- Understand the terms of principal, interest, Time and compound interest.

- Understand the calculation of compound interest when the interest is compounded annually, half-yearly and quarterly.

Introduction:

The addition of interest to the principal is called compounding. Compound interest is the interest on interest. When of interest becomes due at the end of one year, it is added to the principal, instead of being paid over to the lender and this sum becomes the principal for 2nd year. The principal for 2nd year added to the interest on this sum, becomes the principal for the 3rd year and so on. In this case, we say that the money is lent or borrowed at compound interest.

Compound Interest (C.I.) = Amount (A) – Principal(P)

Principal(P): The Money borrowed or lent out for a certain period is called the principal or the sum.

Interest :

Extra money paid for using other's money is called interest.

Compound interest (C .I) : Compound interest is the interest calculated on the principal and the interest accumulated over the previous period. It is different from simple interest, where interest is not added to the principal while calculating the interest during the next period. Compound interest is usually denoted by C.I.

Formulae:

Let Principal = P , Rate = R% per annum, Time = t years

- When interest is compound Annually:

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^n$$

- When interest is compound Half –yearly:

$$\text{Amount} = p \left[1 + \frac{(R/2)}{100} \right]^{2n}$$

- When interest is Compound Quarterly:

$$\text{Amount} = p \left(1 + \frac{(R/4)}{100} \right)^{4n}$$

- When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years

$$\text{Amount} = p \left(1 + \frac{R}{100} \right)^3 \times \left(1 + \frac{\frac{2}{5}R}{100} \right)$$

- If the rate of interest differs from year to year ,i.e. it is r_1 % per annum for 1st year , r_2 % per annum for 2nd

year and r_3 % per annum for 3rd year , then amount after

three years will be : $P\left(1 + \frac{r_1}{100}\right)\left(1 + \frac{r_2}{100}\right)\left(1 + \frac{r_3}{100}\right)$

➤ Present worth of Rs. x due n years hence is given by:

$$\text{Present Worth} = \frac{x}{\left(1 + \frac{R}{100}\right)^n}$$

WORKED OUT PROBLEMS

1. Find compound interest on Rs.7500 at 4%per annum for 2

years, compounded annually.

Ans: Here $P = \text{Rs.}7500$, $R = 4$, $n = 2$ years

Amount =

$$\left[7500 \times \left(1 + \frac{4}{100} \right)^2 \right] = \left(7500 \times \frac{26}{25} \times \frac{26}{25} \right) = \text{Rs.}8112$$

$$\therefore C.I. = \text{Rs.}(8112 - 7500) = \text{Rs.}612$$

2. Find compound interest on Rs. 8000 at 15% per annum for 2 years 4 months, compounded annually.

Ans: Here Time (n) = 2 years 4 months = $2\frac{1}{3}$ years

$$\text{Amount} = \left[8000 \times \left(1 + \frac{15}{100} \right)^2 \times \left(1 + \frac{\frac{1}{3} \times 15}{100} \right) \right]$$

$$= \text{Rs.} \left(8000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{21}{20} \right) = \text{Rs.} 11109$$

3. Find the compound interest on Rs. 10,000 in 2 years at 4% per annum the interest being compounded half – yearly.

Ans: Here P = Rs. 10000 , Rate (R) = 2% per half- yearly ,
Time (n) = 2 years = 4 half – years.

$$\text{Amount} = \left(10000 \times \left(1 + \frac{2}{100} \right)^4 \right) =$$

$$\left(10000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \right) = \text{Rs.} 10824.32$$

$$\text{C.I.} = 10824.32 - 10000 = \text{Rs.} 824.32$$

4. Find the compound interest on Rs. 16000 at 20% per annum for 9 months, compounded quarterly.

Ans: Here P = Rs. 16000 , Time (n) = 9 months = 3 quarters

Rate(R) = 20% per annum = 5% per quarter

$$\text{Amount} = \left(16000 \times \left(1 + \frac{5}{100} \right)^3 \right) = \left(16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \right)$$

$$= \text{Rs.}18522.$$

$$\text{C.I.} = 18522 - 16000 = \text{Rs.}2522$$

5. If the simple interest on a sum of money at 5% per annum for 3 years is Rs.1200, find the compound interest on the same sum for the same period at the same rate.

Ans: Here Rate(R) = 5% per annum , Time = 3 years , S.I. = Rs. 1200.

$$\text{So, } P = \left(\frac{100 \times S.I.}{R \times T} \right) = \left(\frac{100 \times 1200}{5 \times 3} \right) = \text{Rs. } 8000$$

$$\text{Amount} = \left(8000 \times \left(1 + \frac{5}{100} \right)^3 \right) = \left(8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \right)$$

$$= \text{Rs.}9261$$

$$\text{C.I.} = 9261 - 8000 = \text{Rs.}1261$$

6. In what time will Rs. 1000 become Rs.1331 at 10% per annum compounded annually ?

Ans: Here P = Rs. 1000 , Amount = Rs. 1331, Rate = 10% per annum.

Let the time be n years. Then,

$$\left(1000\left(1+\frac{10}{100}\right)^n\right) = 1331$$

$$\left(\frac{11}{10}\right)^n = \left(\frac{1331}{1000}\right) = \left(\frac{11}{10}\right)^3$$

$\therefore n = 3$ years.

7. If Rs. 500 amounts to Rs.583.20 in 2 years compounded annually , find the rate of interest per annum .

Ans: P = Rs.500 , Amount = Rs.583.20 , Time(n)= 2 years

Let the rate be R% per annum . Then,

$$\text{Amount} = \left(P\left(1+\frac{R}{100}\right)^n\right)$$

$$583.20 = \left(500\left(1+\frac{R}{100}\right)^2\right)$$

$$\left(\frac{583.20}{500}\right) = \left(1+\frac{R}{100}\right)^2 \Rightarrow \left(1+\frac{R}{100}\right)^2 = \frac{5832}{5000} = \frac{11664}{10000}$$

$$\therefore \left(1+\frac{R}{100}\right)^2 = \left(\frac{108}{100}\right)^2 \Rightarrow 1+\frac{R}{100} = \frac{108}{100}$$

So , Rate (R)= 8% per annum.

EXERCISE

1. Find the compound interest (CI) on Rs. 12,600 for 2 years at 10% per annum compounded annually.
2. At what rate of compound interest per annum, a sum of Rs. 1200 becomes Rs. 1348.32 in 2 years?
3. A TV was bought for Rs. 21,000. The value of the TV was depreciated by 5% per annum. Find the value of the TV after 3 years. (Depreciation means the reduction of value due to use and age of the item.)
4. Find the compound interest on Rs 48,000 for one year at 8% per annum when compounded half-yearly.
5. Find the compound interest on Rs. 8000 at 15% per annum for 2 years 4 months, compounded annually.

Answers:

1. Rs.2646 2. $R = 6\%$ 3. Rs. 18,004.875 4. Rs. Rs 3,916.80 5. Rs. 11,109



UNIT-III

DATA INTERPRETATION

Tabulation
Bar Graphs
Pie Charts
Line Graphs
Venn Diagrams

3.1 TABULATION

LEARNING OUTCOMES:

Upon practicing the problems on tabulation students are able to:

1. present the given data in a simplified and meaningful manner.
2. analyses the data through comparison
3. establish correlation among various items involved and draw valid conclusions.

Introduction

Tabulation is a basic data analysis tool. It is systematic and logical representation of data in horizontal and vertical cells. The horizontal arrangements are called rows whereas the vertical arrangements are called columns of the table. The main purpose of tabulation is to improve the readability of data, compare various items and sub-items involved, establish logical relation among them and drew valid conclusions.

The main components of a table are:

Table number: Table number is helpful for easy identification and further reference. It is to be mentioned at the top of the table.

Title of the table: Title describes the main content represented in the table. It should be clear and concise.

Head note: It gives information about units of contents and usually given besides are below the Title of the table in brackets.

Row/Column headings: These are titles to describe the contents of each row and column of the table. The units of the contents, if required, are also to be mentioned along with row/column headings.

Body of the table: The information entered in the rows/sub-rows and columns/sub-columns of the table is usually called as its body.

Foot note: It is a simple statement indicating the source of the data given in the table. We can also specify any exceptions or need of special examination of some items in the table in the foot note.

Example-1: From the 2019-20 annual reports of XYZ college it is found that, the college has been offering three-year under graduate programs in Arts, Commerce and Science disciplines. In that academic year, the total student strength of

the college is 792 out of which 252 are third year students and 264 are second year students. In each year one-third of the total number of students belongs to arts stream, whereas the students of commerce stream amounts to 100, 95 and 90 in the first, second and third years respectively. Further it is observed that, in each year half of the arts students, two-fifth of the commerce students and two-third of the science students are girls and the remaining are boys. Tabulate the above data and answer the following questions.

- 1) In the given academic year what are the total number of girls studying in the college?
- 2) How many first-year students are boys in the academic year 2019-20?
- 3) What is the percentage of arts students in the total student strength of given academic year?
- 4) What is the difference between number of boys and girls in the academic year 2019-20 when all years and all programs put together?
- 5) What is the total number of boys in second year?

Solution:

First let us find the number of first year students = $792 - (264 + 252) = 276$

Now let us find the number of arts students of each year

Number of arts students in first year = $\frac{1}{3}$ (Total first year students) = $\frac{1}{3}$ of 276 = 92

Out of these 92 students, half of them girls and remaining half are boys

Therefore, in the arts group, the number of first year girls is 46 and boys also 46.

The number of girls and boys of arts group for the second and third years can be filled in a similar way.

Now each year the number of girls of commerce group = $\frac{2}{5}$ of the total commerce students of that year and remaining $\frac{3}{5}$ are boys

The number of first year girls in the commerce group = $\frac{2}{5}$ of 100 = 40 and

The number of first year boys in the commerce group = 60

Similarly, in the second year there are $\frac{2}{3}$ of 95 = 38 girls and the remaining 57 are boys in commerce group. In the third year there will be 36 girls and 54 boys in commerce stream.

In science group, the total first year students = $276 - (92+100)$
= 84

In science group the number of first year girls = 56 ($\frac{2}{3}$ of 84)
and boys = 28 (remaining $\frac{1}{3}$)

The number of second and third year girls of science group are 54 and 52 respectively whereas, the number of boys for the respective years are 27 and 26.

All the above information can be represented in a table form as given below:

TABLE-1

Title: Student strength of XYZ college in the academic year 2019-20

	Arts		Commerce		Science		
	Boys	Girls	Boys	Girls	Boys	Girls	Total
I year	46	46	60	40	28	56	276
II year	44	44	57	38	27	54	264
III year	42	42	54	36	26	52	252
Total	132	132	171	114	81	162	792

Source: XYZ College annual reports 2019-20

Now using the organized data in a table form it is easy to answer the given questions.

- 1) The total number of girls in the college in 2019-20 = $132+114+162 = 408$.

2) The number of first-year boys in the academic year 2019-20 = $46+60+28 = 134$.

3) The percentage of arts students in the total student strength of 2019 = $(264/792) \times 100$

$$= (1/3) \times 100 = 33\frac{1}{3}\%$$

4) The difference between number of boys and girls in the academic year 2019-20

$$= (132+114+162) - (132+171+81)$$

$$= 408 - 384 = 24.$$

5) The total number of boys in second year = $44+57+27 = 128$.

Example-2: Study the following table carefully and answer the following questions.

Number of paddy bags produced by 5 farmers over the years						
Years	Farmers					
	A	B	C	D	E	F
2000	420	360	396	528	492	444
2001	564	492	576	612	576	540
2002	588	612	624	648	576	564
2003	600	660	648	636	612	600
2004	648	708	684	672	660	672
2005	732	744	720	756	708	720

1) What is the average number of paddy bags produced by Farmer D over the years?

- A) 624 B) 642 C) 674 D) 672
 E) None of these

2) The number of paddy bags produced by Farmer B in the year 2003 are approximately what percent of the paddy bags produced by Farmer B over the years?

- A) 24 B) 18 C) 21 D) 3 E) 15

3) What is the respective ratio of the paddy bags produced by Farmer A to the Farmer E in the year 2005?

- A) 60 : 61 B) 61 : 63 C) 60 : 63 D) 61 : 59

E) None of these

4) What is the respective ratio of the paddy bags produced by Farmers A, B and C together in the year 2000 to the paddy bags produced by Farmers D, E and F together in the same year?

- A) 49 : 64 B) 51 : 61 C) 49 : 61 D) 51 : 55

E) None of these

5) What is the difference between the total number of paddy bags produced by all the farmers together in the year 2001 and the year 2004?

- A) 684 B) 648 C) 846 D) 864
E) None of these

Solution:

- 1) Average number of paddy bags produced by Farmer D

$$= \frac{(528+612+648+636+672+756)}{6} = 642.$$

- 2) Percent of paddy bags produced by Farmer B in the year 2003 to the paddy bags produced by Farmer B over the years

$$= \frac{660}{(360+492+612+660+708+744)} \times 100 = 18.45\%$$

=18% approximately

- 3) Ratio of the paddy bags produced by Farmer A to the Farmer E in the year 2005

$$= 732 : 708$$

$$= 183 : 177 \text{ (cancelling 4 in both of the terms)}$$

$$= 61 : 59 \text{ (cancelling 3 in both of the terms)}$$

4) Ratio of the paddy bags produced by Farmers A, B and C together in the year 2000 to the paddy bags produced by Farms D, E and F together in the same year

$$= (420+360+396) : (528+492+444)$$

$$= 1176 : 1464$$

$$= 49 : 61 \text{ (cancel 24 in both of the terms)}$$

5) The total number of paddy bags produced by all the farmers together in the year 2001 is 3360 and in the year 2004 is 4044.

The difference of paddy bags produced in these two years
 $= 4044 - 3360 = 684.$

Problem-1: Study the following table to answer the questions that are given below it.

Expenditures of a company per annum over the given years
(in lakh rupees)

Items of Expenditure →	Salary	Fuel and Transport	Bonus	Interest on loans	Taxes
Year ↓					
1998	288	98	3.00	23.4	83
1999	342	112	2.52	32.5	108
2000	324	101	3.84	41.6	74
2001	336	133	3.68	36.4	88
2002	420	142	3.96	49.4	98

1) The ratio between the total expenditure on taxes for all the years and the total expenditure on fuel and transport for all the years respectively is approximately:

A) 4 : 7 B) 10 : 13 C) 15 : 18 D) 5 : 8 E) 2 : 3

2) The total expenditure of the company over these items during the year 2000 is (in lakhs of rupees):

A) 544.44 B) 501.11 C) 446.46 D) 478.87
E) 612.13

3) What is the average amount of interest per year which the company had to pay during this period (in lakhs of rupees)?

- A) 32.43 B) 33.72 C) 34.18 D) 35.69
E) 36.66

4) Total expenditure on all these items in 1998 was approximately what percent of the total expenditure in 2002?

- A) 62% B) 66% C) 69% D) 71% E) 73%

5) The total amount of bonus paid by the company during the given period is approximately what percent of the total amount of salary paid during this period?

- A) 0.1% B) 0.5% C) 1% D) 1.25% E) 1.11%

Problem-2: The following table gives sales of different bulbs manufactured by a company over the years. Read the table and answer the following questions

Sales of bulbs by a company over years (in thousands)

Years	Types of bulbs				
	5 watts	7watts	9watts	12 watts	15 watts
1992	108	144	102	75	114
1993	126	126	84	90	102
1994	135	114	105	96	75
1995	75	90	90	105	150
1996	90	75	75	90	135
1997	120	60	45	105	165
1998	145	55	100	115	160

- 1) The total sales over the years are highest for which type of bulb?
- 2) How many more 9 watts bulbs were sold in 1998 compared to 1996?
- 3) Which type of bulbs shows continuous decrease in sales?

- 4) What is the percentage of decrease in the sales of 9 watt bulbs from 1994 to 1996?
- 5) Approximately what percentage of sales is decreased in case of 7 watt bulbs from 1991 to 1998?

Answers:

Problem-1: 1-B, 2-A, 3-E, 4-C, 5-C

Problem-2: 1-15watt bulbs, 2-25000, 3-7 watt, 4-40%, 5-62%

Web Resources:

<https://www.allindiaexams.in/reasoning/data-interpretation/tabulation>

<https://testbook.com/objective-questions/mcq-on-tabulation--5eea6a1639140f30f369f559>



3.2.BAR DIAGRAM

LEARNING OUTCOMES:

After completion of this topic, student will be able to

- ❖ Identify and define types of bar graphs
- ❖ Describe the features and benefits of bar graph
- ❖ Compare sets of data between groups at a glance
- ❖ Easily summarize and compare values within a data category

INTRODUCTION

In previous chapters we studied, how to arrange the given data in tabular form. However, the tabular representation of data is not much interesting and in fact it is in confusion status. But now, it is upgraded to diagrammatic representation of data, which is more effective and, in which the large data can be converted to a simple form. There are three types of diagrammatic representations, which are geometric, Frequency and Arithmetic. In Geometric form, we have Bar diagrams and Pie diagrams. To answer the questions related

to bar graphs, we must aware of averages, ratios and percentages as studied in previous chapters.

BAR DIAGRAM

A Bar graph is a graph with rectangular bars with the lengths proportional to the values that they represent. Bar graphs are used for comparing two or more values. The bars can be oriented as vertically or horizontally. The bar gives a visual display for comparing quantities in different categories.

BAR AND ITS FEATURES

- All rectangular bars should have equal width and with different length.
- The rectangular bars must be on a common base.
- The rectangular bars should be placed in equidistance.
- The rectangular bars can be oriented as vertically or horizontally.

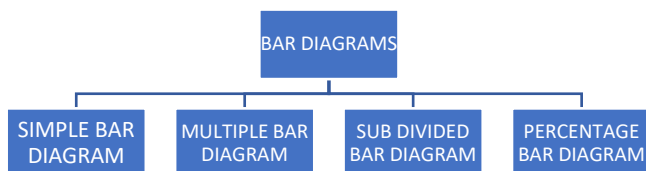
DESCRIBING THE BAR CHART

To answer the given question of Bar graph, we have to describe the chart as given below

- ❖ **Introduction:** The introduction should be the message about what bar graph is talking about actually.
- ❖ **Overview:** Describe in brief with 2- 4 key features. which does not contain too many numbers, percentages and dates. Search for the biggest, smallest or unchanged part and write briefly about the changes.
- ❖ **Feature 1:** Sector that experiences increase, decrease or unchanged.
- ❖ **Feature 2:** Sector that experiences increase, decrease or unchanged. And so on.

5.TYPES OF BAR DIAGRAM

There are four types of Bar diagrams.

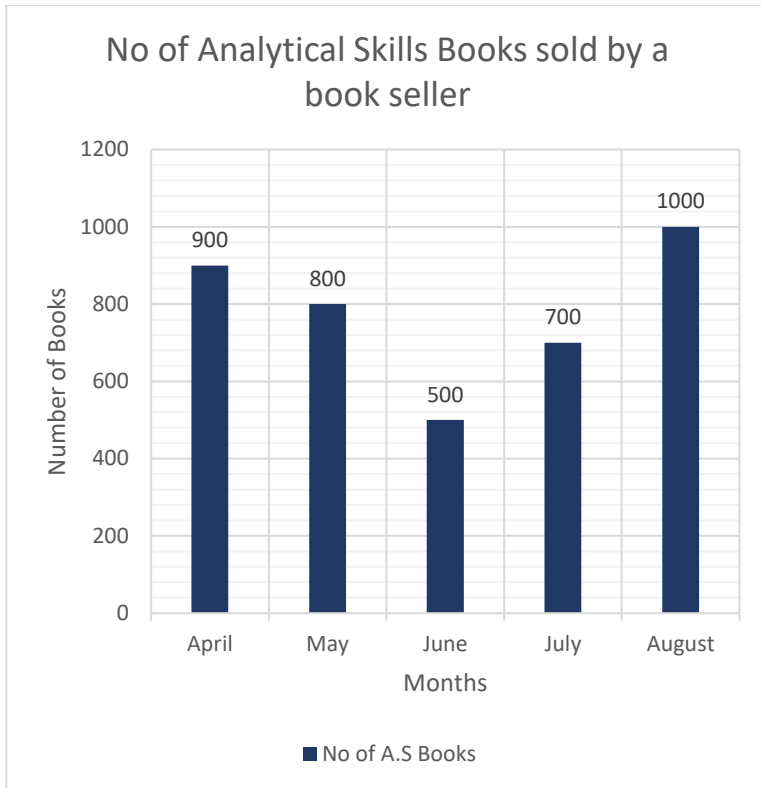


In this chapter we discuss only simple bar diagrams and multiple bar diagrams

SIMPLE BAR DIAGRAM

Simple bar diagrams are those diagrams, which are based on a single set of numerical data like birth rate in some consecutive years, pass percentage of students in some years, No. of admissions in some consecutive years etc. Here is an example of simple bar diagram

Problem: The bar graph shows the number of copies “Analytical skills” sold by a book seller during the period from April’2021 to August’2021



Description:

Introduction: The graph mainly talks about the no of books sold by the book seller in consecutive five months in the year 2021

Over view: The books sold by the seller are gradually decreased from April to June. After June number is increased up to August.

Feature : In April month books sold is 900, similarly in May-800, June-500, July-700 and August-1000

Question and Answers:

1.What is the difference of the sale of books in April and August?

A: It is clearly we can observe that $1000-900=100$ Books

2.In which month was the sale of the book maximum?

A: It clearly visible from graph is August.

3.In which month was the sale of the book minimum?

A: It clearly visible from graph is June.

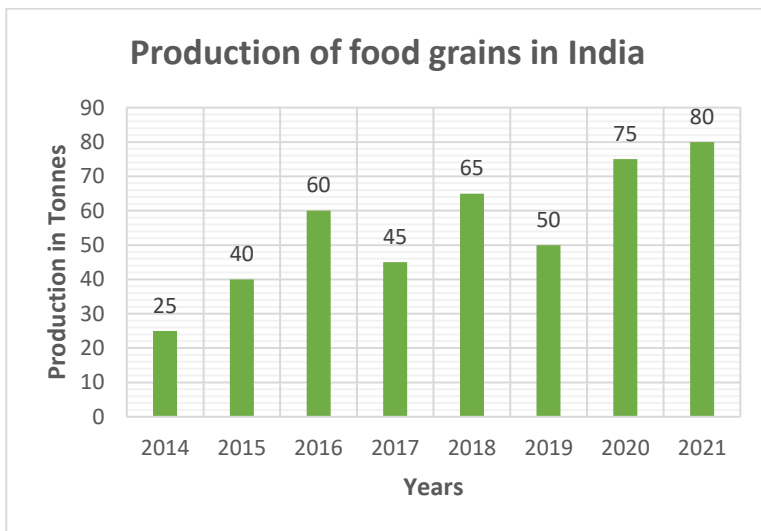
4. What is the total sale of the books during these five months?

A: $900+800+500+700+1000=3900$ books

5. What is the average sale of the books during these five months?

A: $\frac{\text{Total books sold by seller}}{\text{Number of months}} = \frac{3900}{5} = 780 \text{ books}$

Problem : The following bar graph shows the production of food grains by a company in “Tonnes” over the years. Study the graph carefully and answer the questions given below.



Description :

Introduction : The graph mainly talks about the production of food grains by a company during the years 2014 to 2021

Over view: Production of food grains is not in decreasing mode and not in increasing mode but it is in oscillating mode.

Feature : Let's draw a table

Years	2014	2015	2016	2017	2018	2019	2020	2021
Production	25 Tonnes	40 Tonnes	60 Tonnes	45 Tonnes	65 Tonnes	50 Tonnes	75 Tonnes	80 Tonnes

Total production is 440 tonnes

Question and Answers :

1.What was the decline % in the production of food grains from 2016 to 2017 ?

A: It is clearly visible from table that production in 2016 was 60 tonnes and in 2017 was 45 tonnes

$$\% \text{ Decrease} = \frac{\text{Original value} - \text{decreased value}}{\text{original value}} = \frac{60 - 45}{60} \times$$

$$100 = \frac{15}{60} \times 100 = 25\%$$

2.What is the ratio between the production of food grains in the year 2019 to that of 2016 ?

A: $\frac{\text{Production of food grains in 2019}}{\text{Production of food grains in 2016}} = \frac{50}{60} = \frac{5}{6}$

$\therefore$ The ratio is 5:6

3.What is the increase % in the production of food grains in 2021 as compared to that in 2014 ?

A: As per the table Production of food grains in 2021 was 80 tonnes and in 2014 was 25 tonnes

$$\begin{aligned}\% \text{ Increase} &= \frac{\text{Increase value} - \text{Original value}}{\text{original value}} = \frac{80-25}{25} \times 100 = \\ &\frac{55}{25} \times 100 = 220\%\end{aligned}$$

4.In how many years was the production of food grains more than average production of the given years ?

A: Now Average production = $\frac{\text{Total production}}{\text{Number of years}} = \frac{440}{8} = 55$ tonnes

It is clearly visible from the table that more than 55 tonnes production is in years 2016,2018,2020,2021

∴ 4 years.

5.What is the difference between the production of food grains in 2016 and 2019 ?

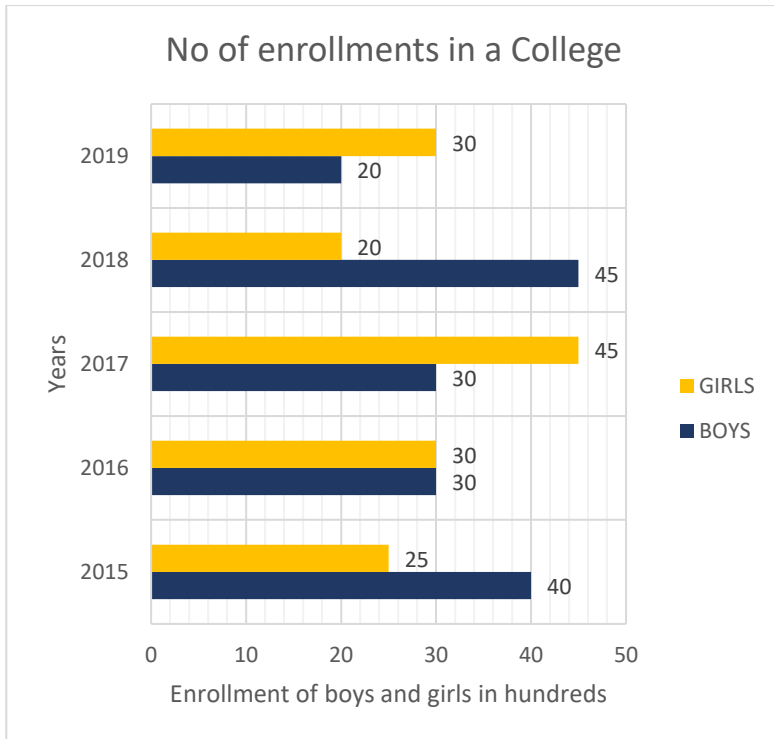
A: Difference = Production of food grains in 2016 –
Production of food grains in 2019

$$= 60-50 = 10 \text{ tonnes}$$

MULTIPLE BAR DIAGRAM

Multiple bar diagrams are those diagrams which shows two or more sets of data simultaneously. Generally, these diagrams are used to make comparison among the sets of data i.e more independent variables and one dependent variable. For example production of different companies in different years, no of enrollments in different groups in different years etc . Here are examples of multiple bar diagram

Problem : Study the bar graph carefully and answer the questions given below.



Description:

Introduction: The graph mainly talks about the enrollment of boys and girls in hundreds of a college during the years 2015 to 2019

Over view: The enrollment of boys and girls seems to be up and down in number. In 2016 enrollment seems to be equal in both cases.

Feature: Let's draw a table

Years	2015	2016	2017	2018	2019	Total
Boys	4000	3000	3000	4500	2000	16500
Girls	2500	3000	4500	2000	3000	15000

Question and Answers:

- 1. In which year enrollment of Boys and Girls is equal?**

A: 2016 (It is clearly visible in graph)

- 2. What is the ratio of total enrollment of boys to girls?**

$$\text{A: } \frac{\text{Total enrollment of Boys}}{\text{Total enrollment of Girls}} = \frac{16500}{15000} = \frac{33}{30}$$

33:30

- 3. What is the average of total enrollments in five years?**

A: Average

$$\begin{aligned} &= \frac{\text{Total enrollment of Boys} + \text{Total enrollment of Girls}}{\text{Number of Years}} = \\ &\frac{16500 + 15000}{5} = \frac{31500}{5} = 6300 \end{aligned}$$

4. Find the ratio of girls enrollment in 2015 to the boys enrollment in 2018 ?

$$\text{A: } \frac{\text{Girls enrollment in 2015}}{\text{Boys enrollment in 2018}} = \frac{2500}{4500} = \frac{5}{9}$$

5:9

5. Find the no of times for which the enrollment of girls was higher than the average enrollment in that year ?

A: First of all we have to find the average enrollment in each year

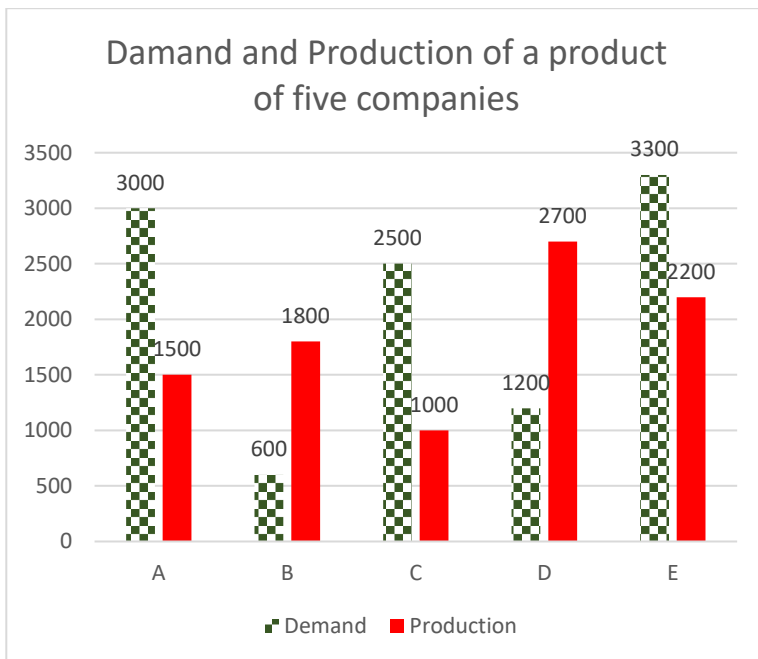
Years	2015	2016	2017	2018	2019
Boys	4000	3000	3000	4500	2000
Girls	2500	3000	4500	2000	3000
Average	3250	3000	3750	3250	2500

It is clearly visible in the table that in 2017 and 2019 ,there was a higher enrollment of girls than the average of that years.

∴ 2 times.

Problem : Study the bar graph carefully and answer the questions given below.

Demand and production of a product of five companies A, B, C, D and E for November 2015



Description:

Introduction: The graph mainly talks about the demand and production of a product of five companies A, B, C, D and E

Over view: Demand of product is high than production in companies A, C and E, where as it is reverse in companies B and D

Feature: Let's draw a table

Company	A	B	C	D	E	Total
Demand	3000	600	2500	1200	3300	10600
Production	1500	1800	1000	2700	2200	9200

Question and Answers:

1. What percentage of the demand of company C is to that of the company B ?

A: $\frac{\text{Demand of company B}}{\text{Demand of company C}} \times 100 = \frac{600}{2500} \times 100 = 24 \%$

2. What is the difference between the average demand and average production of the five companies taken together?

$$\text{A: Average demand} = \frac{10600}{5} = 2120$$

$$\text{Average production} = \frac{9200}{5} = 1840$$

$$\text{Difference} = 2120 - 1840 = 280$$

3. The average production of the companies A and B and that of the companies D and E are in the ratio is -----

$$\begin{aligned} \text{A: Average production of A and B} &= \frac{1500+1800}{2} = \\ \frac{3300}{2} &= 1650 \end{aligned}$$

$$\begin{aligned} \text{Average production of D and E} &= \frac{2700+2200}{2} = \\ \frac{4900}{2} &= 2450 \end{aligned}$$

$$\text{Ratio} = \frac{1650}{2450} = \frac{33}{49}$$

33:49

4. What is the ratio of companies having more demand the production to those having more production than demand?

A: Ratio

$$= \frac{\text{Number of companies having more demand than production}}{\text{Number of companies having more production than demand}} = \frac{3}{2}$$

3:2

5. How many times is the production of company D of that of the company A?

A: Production of a company A =1500

Production of a company D = 2700

$$\text{The required times} = \frac{2700}{1500} = 1.8$$

SUB DIVIDED BAR DIAGRAM

A sub divided bar diagram is used to present data having 2 or more components. Sub divided or component bar chart is used to represent data in which the total magnitude is divided into different components. Such bar diagrams also called Composite bar chart and stacked bar diagrams.

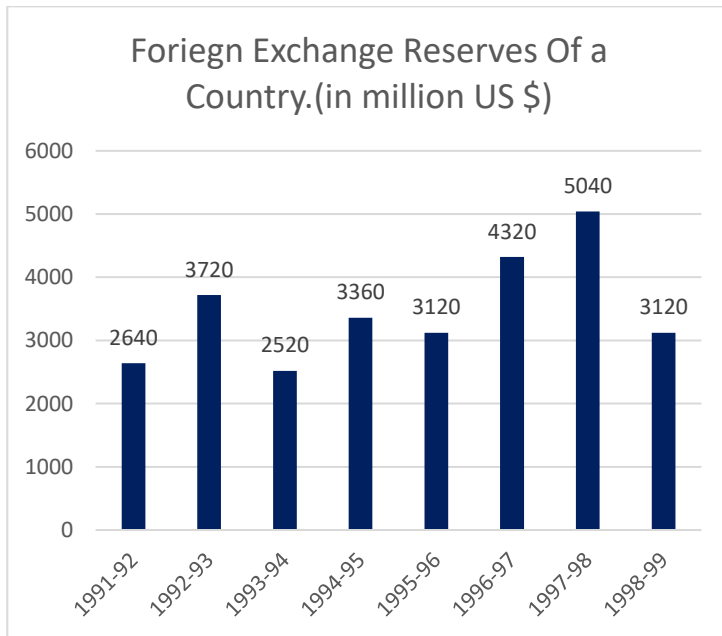
PERCENTAGE BAR DIAGRAM

Percentage bar diagrams are those diagrams, which simultaneously different parts of the values of a set of

data in terms of percentage. In this diagram each bar is taken total value as 100. Each part of a bar is shown as a part of 100. Actually it is the percentage form of sub divided bar diagram

EXERCISE

1. The bar graph given below shows the foreign exchange reserves of a country (in million US \$) from 1991-1992 to 1998-1999. Observe the graph and answer the questions given below.

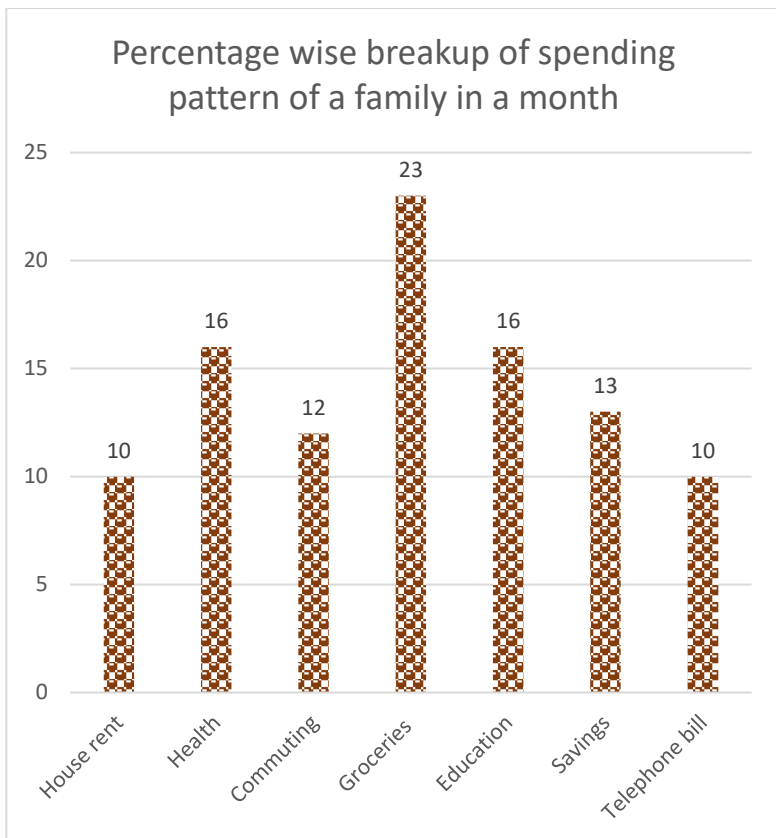


1. The ratio of the number of years, in which the foreign exchange reserves are above the average reserve, to those in which the reserves are below the average reserves is?
2. The foreign exchange reserves in 1997-98 was how many times that in the year 1994-95?
3. . For which year, the present increase of foreign exchange reserves over the previous year, is the highest?
4. . The foreign exchange reserves in 1996-97 were approximately what percentage of the average foreign exchange reserves over the period under review?
5. What was the percentage increase in the foreign exchange reserves in the year 1997-98 over the year 1993-94?

ANSWERS

1. 3:5 2. 1.5 3.1992-93 4. 125% 5 .100

2. Study the bar chart carefully to answer the questions given below.



Total amount spent in a month = Rs 60,000

1. The amount spent by the family on commuting is ----
-?
2. The total amount spent by the family on telephone bills, health and education together is -----?

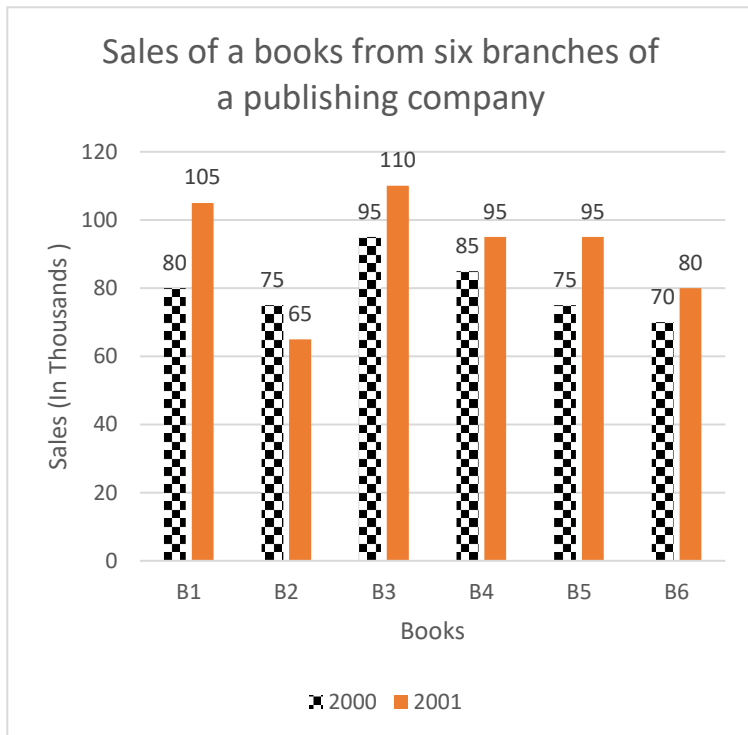
3. What is the ratio of the amount spent by the family on groceries to the amount spent on house rent?
4. Amount invested by the family in savings forms, what percentage of the amount spent on health?
5. Total amount spent by the family on commuting and telephone bills together forms what percent of the amount spent on groceries?

ANSWERS:

1. Rs 7200 2. Rs 25200 3. 23:10 4. 81.25% 5. 91.65%

3. The bar graph given below shows the sales of books (in thousands) from six branches of a publishing company during two consecutive years 2000 and 2001. Sales of books (in thousands) from six branches – B1,B2,B3,B4,B5 and B6

of a company in 2000 and 2001



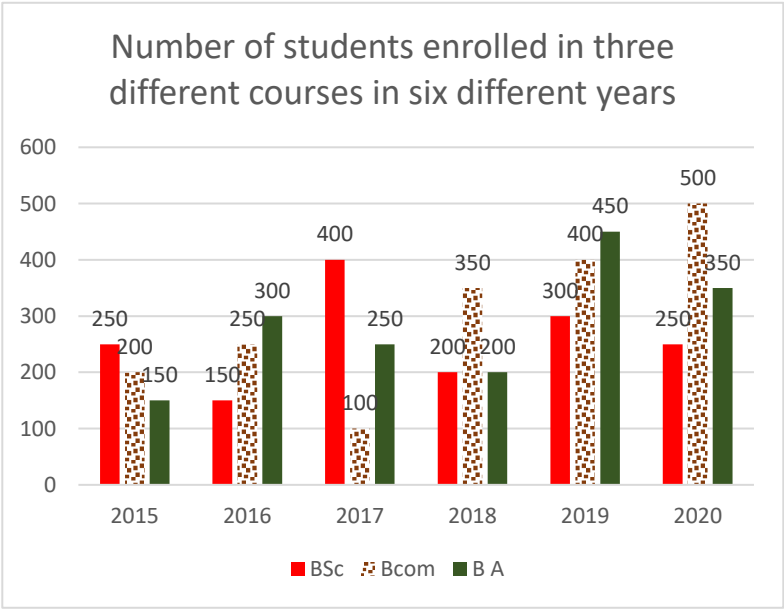
1. What is the ratio of the total sales of branch B2 for two years to the total sales of branch B4 for two years?
2. Total sales of branch B6 for both years is what percent of the total sales of branch B3 for both the years?
3. What percentage of average sales of branches B1, B2 and B3 in 2001 is the average sales of branch B1, B3 and B6 IN 2000?

4. What is the average sales of the all branches for the year 2000?
5. Total sales of branches B1, B3 and B5 together for both years is?

ANSWERS:

1. 7:9 2. 73.17% 3. 87.5% 4. 80 5. 560

4. Study the bar chart carefully to answer the questions given below.



1. In which year was the difference between the students enrolled in BSc and B.com the highest?
2. What was the percentage increase in the number of students enrolled in B.com in the year 2019 as compared to previous year?
3. Number of students enrolled in BSc in the year 2015 and 2019 together was approximately, what percentage of the total number of students enrolled in B A over all the years together?
4. What was the average number of the students enrolled in B.com over all the years together?
5. What was the respective ratio between the number of students enrolled in B A in the year 2020 and the number of students enrolled in BSc in the year 2017?

ANSWERS

- | | | |
|----------------|---------------|---------------|
| 1. 2017 | 2. 50% | 3. 32% |
| 4. 300 | 5. 7:8 | |

REFERENCES

1. https://en.wikipedia.org/wiki/Bar_chart#:~:text=A%20bar%20chart%20or%20bar,sometimes%20called%20a%20column%20chart.

2. <https://youtu.be/LEM2xQEVYGo>
3. <https://youtu.be/O5eb1uWsLrU>
4. <https://youtu.be/KIpGdKVBhOI>



3.3 PIE DIAGRAM

LEARNING OUTCOMES:

At the end of this topic, student will be able to

- ❖ Identify the characteristics of a pie chart
- ❖ Compare each groups contribution to the whole
- ❖ Interpret data presented in pie charts
- ❖ Calculate frequencies of a pie chart, when given the sample size

INTRODUCTION

In previous topic we discussed about description of bar graph and how to solve problems related to bar graph. Similar to the bar graph we have another type of diagrammatic representation of data, which is most effective way to understand data and it is called pie diagram. A pie chart is a type of a chart that visually displays data in a circular way. They are very widely used in the business world and the mass media.

PIE DIAGRAM

A pie chart (or a circular chart) is a circular statistical graph, which is divided into slices to illustrate numerical proportion. In a pie chart, the arc length of each slice is proportional to the quantity it represents i.e. the slice of pie shows the relative size of the data. The entire pie represents 100% of the data set, while each sector represents a portion of the whole data set. The portion that exists in the pie are measured as an angle of the total 360 degrees.

In this topic we will discuss the pie charts based on percentage values and degree values

BASIC MATH TO LEARN

As we learn in previous topics some basic mathematics are very important to answer the questions regarding pie diagram

❖ **Basic mathematical operations** like addition, subtraction, multiplication and division

❖ **Ratio** i.e. $a:b = \frac{a}{b}$

❖ **Percentage:** Recall two important formulae

1. X is what % of Y $= \frac{X}{Y} \times 100$

2. X is what % more or less than Y $= \frac{X-Y}{Y} \times 100$

❖ **Average** $= \frac{\text{Sum of quantities}}{\text{Number of quantities}}$

❖ **Percentage $\Leftrightarrow$ Degree**

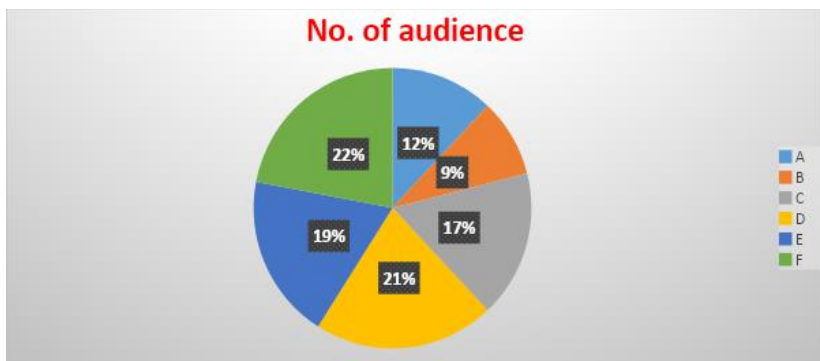
- $100\% = 360^0$
- $50\% = 180^0$
- $25\% = 90^0$
- $20\% = 72^0$
- $10\% = 36^0$
- $2\% = 7.2^0$
- $1\% = 3.6^0$

PIE DIAGRAM BASED ON PERCENTAGE VALUES

A pie chart uses percentages to compare information. Percentages are used to represent a whole. The whole is equal to 100%.

WORKED OUT PROBLEMS:

1.The following pie diagram shows the number of audiences in cinema theaters A, B, C, D, E and F. Total number of audience is 4800



DESCRIPTION: Total number of audience = 4800 = 100%

❖ $50\% = 2400$

❖ $10\% = 480$

❖ $1\% = 48$

Draw a table

Name of theater	A	B	C	D	E	F
Percentage of audience	12%	9%	17%	21%	19%	22%

QUESTION AND ANSWERS

1. What was the average number of audience in theater A, theater C and theater F?

A: Theater A + Theater C + Theater F = 12% + 17% + 22% = 51%

51% of total audience = 51% of 4800

$$= (50\% + 1\%) \text{ of } 4800$$

$$= 50\% \text{ of } 4800 + 1\% \text{ of } 4800$$

$$= 2400 + 48 = 2448$$

$$\text{Average} = \frac{2448}{3} = 816.$$

2. If the cost of one ticket is Rs100/-, what was the total amount paid by the audience in theater B?

A: Total number of audience in theater B = 9% of 4800

$$= (10 - 1) \% \text{ of } 4800$$

$$= 10\% \text{ of } 4800 - 1\% \text{ of } 4800$$

$$= 480 - 48 = 432$$

∴ Total amount paid by the audience in theater B = $432 \times 100 = 43200$

Rs 43200/-

3. Number of audience in theater E is approximately what percentage of the total audience in theater B and theater D?

A: X is what % of Y = $\frac{X}{Y} \times 100$

Audience in theater E is approximately what percentage of the total audience in theater B and D =

$$\frac{\text{Theater E}}{\text{Theater B} + \text{Theater D}} \times 100 = \frac{19\% \text{ of } 4800}{(21+9)\% \text{ of } 4800} \times 100$$

$$= \frac{19\% \text{ of } 4800}{30\% \text{ of } 4800} \times 100 = \frac{19}{30} \times 100 = \frac{190}{3} = 63.33\%$$

4. What is the difference between the number of audience in theater C and the number of audience in theater A?

A: Difference = Number of audience in theater C – Number of audience in theater A

$$= (17\% - 12\%) \text{ of } 4800$$

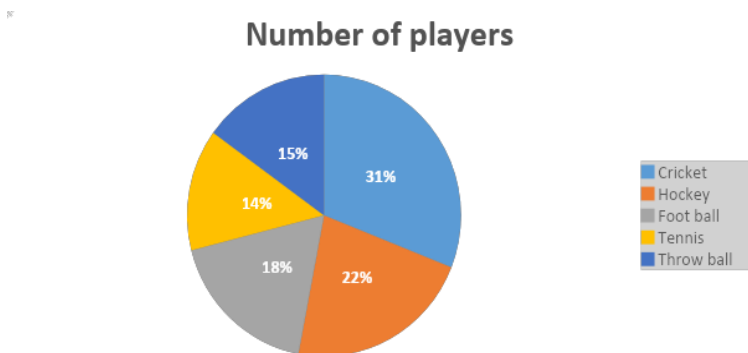
$$= 5\% \text{ of } 4800$$

$$= 240 \text{ (clearly observed from description)}$$

5. What is the total number of audiences in theater D, theater E and theater F?

$$\begin{aligned}
 \text{A: Total} &= \text{Theater D} + \text{Theater E} + \text{Theatre F} \\
 &= (21\% + 19\% + 22\%) \text{ of } 4800 \\
 &= 62\% \text{ of } 4800 = (50\% + 10\% + 2\%) \text{ of } 4800 \\
 &= 2400 + 480 + 96 = 2976
 \end{aligned}$$

2. Study the following pie chart carefully. Pie chart shows the number of players = 3000



DESCRIPTION:

Total number of audience = 3000 = 100%

❖ $50\% = 1500$

❖ $10\% = 300$

❖ $1\% = 30$

Draw a table

Name of game	Cricket	Hockey	Football	Tennis	Throw ball
Percent age of players	31%	22%	18%	14%	15%

QUESTION AND ANSWERS

1.How many number of players in Cricket?

A: Number of players in Cricket = 31% of 3000

$$= 30\% + 1\% = 900 + 30$$

$$= 930$$

2.How many players in Throw ball?

A : Number of players in Throw ball = 15% of 3000

$$= \frac{30\% \text{ of } 3000}{2} = \frac{900}{2} = 450$$

3.What is the difference between the number of players in Football and Tennis?

A: Difference = Number of players in Football – Number of players in Tennis

$$= 18\% \text{ of } 3000 - 14\% \text{ of } 3000$$

$$= 4\% \text{ of } 3000$$

$$= 30 \times 4 = 120$$

4. What is the average of players who are playing Tennis, Throw ball and Cricket?

A: Total players = Tennis + Throw ball + Cricket

$$= 14\% + 15\% + 31\% = 60\% \text{ of } 3000$$

$$= 10\% \times 6 = 300 \times 6 = 1800$$

$$\text{Average players} = \frac{1800}{3} = 600$$

5. How many players are there in Football and Hockey together?

A: Total players = Football + Hockey

$$= 18\% + 22\% = 40\% \text{ of } 3000$$

$$= 10\% \times 3 = 300 \times 3 = 900$$

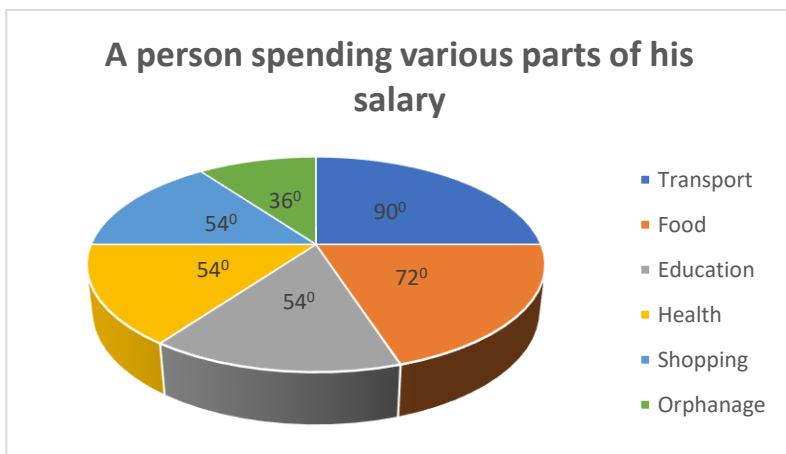
PIE DIAGRAM BASED ON DEGREE VALUES

: In this type, the values of different parts of the individual are given in terms of degree. The total of all the given items is equal to 360° and the degree of angles representing different items are calculated proportionally.

WORKED OUT PROBLEMS:

1. Study the following pie chart carefully.

Salary of a person spending in various parts. Total salary is 60,000



DESCRIPTION:

Draw a table

Type of spending	Transport	Food	Education	Health	Shopping	Orphanage
Degree of spending	90°	72°	54°	54°	54°	36°

Total Salary = 60,000 = 360⁰ = 100%

❖ 50% = 180⁰ = 30,000

❖ 25% = 90⁰ = 15,000

❖ 12.5% = 45⁰ = 7,500

❖ 10% = 36⁰ = 6000

❖ 5% = 18⁰ = 3000

❖ 1% = 3.6⁰ = 600

QUESTION AND ANSWERS

1. What is the difference between the amount spent on transport and the amount spent on Education?

A: As per the observation in description

The amount spending on transport = 90⁰ = 15,000

$$\begin{aligned}\text{The amount spending on education} &= 54^0 = 90^0 - 36^0 \\ &= 15,000 - 6,000 \\ &= 9,000\end{aligned}$$

Difference = 15000 - 9000 = 6000

2. What is the total amount spent on Food, Education and Shopping together?

$$\begin{aligned}\text{A: The amount spending} &= \text{Food} + \text{Education} + \text{Shopping} \\ &= 72^0 + 54^0 + 54^0 \\ &= 180^0\end{aligned}$$

$$= 30000$$

3.What is the average amount spent on Food, Education and Health?

A: Total amount spending == Food + Education + Health

$$= 72^0 + 54^0 + 54^0$$

$$= 180^0$$

$$= 30000$$

$$\text{Average} = \frac{30,000}{3} = 10,000.$$

4.What is the ratio of the amount spent on transport to the amount spent on orphanages?

A: The amount spending on transport : The amount spending on orphanage

$$\frac{90^0 \text{ of } 60,000}{36^0 \text{ of } 60,000} = \frac{15,000}{6000} = \frac{5}{2}$$

5:2

5.What is the amount spent on Shopping?

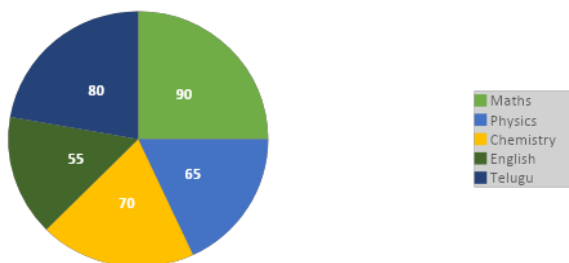
A: The amount spending on shopping = $54^0 = 90^0 - 36^0$

$$= 15,000 - 6,000$$

$$= 9,000$$

2.Study the following pie chart carefully. It shows the total marks obtained by a student is 540

Marks obtained by a student

**DESCRIPTION:**

Draw a table

Subject s	Mathem atics	Physics	Chemis try	English	Telugu
Marks	90 ⁰	65 ⁰	70 ⁰	55 ⁰	80 ⁰

Total Marks = 540 = $360^0 = 100\%$

❖ $50\% = 180^0 = 270$

❖ $25\% = 90^0 = 135$

❖ $10\% = 36^0 = 54$

❖ $5\% = 18^0 = 27$

❖ $1\% = 3.6^0 = 5.4$

QUESTION AND ANSWERS

1. In which subject student obtain 105 marks?

A: $360^0 = 540$

$$x = 105^0$$

By taking cross multiplication we get $540 \times x = 360 \times 105$

$$x = \frac{360 \times 105}{540} = \frac{36 \times 105}{54} = \frac{2 \times 105}{3} = 2 \times 35 = 70$$

$x = 70^0$, From pie chart it is easy to observe that in chemistry student got 105 marks

Ans: Chemistry

2.What is the difference between the marks obtained in Mathematics and Chemistry?

A: Mathematics marks = $90^0 = 135$

Chemistry marks = $70^0 = 105$ (From above answer)

$$\text{Difference} = 135 - 105 = 30$$

3.What percent of the marks obtained in Telugu approximately?

A: Marks obtained in Telugu = 80^0

$$360^0 = 100\%$$

$$80^0 = x$$

By taking cross multiplication we get $360 \times x = 100 \times 80$

$$x = \frac{100 \times 80}{360} = \frac{10 \times 80}{36} = \frac{200}{9} = 22.22\%$$

4. What is the ratio of marks obtained in mathematics to the marks obtained in chemistry?

A: $\frac{\text{Marks obtained in mathematics}}{\text{Marks obtained in chemistry}} = \frac{90}{70} = \frac{9}{7}$

9:7

5. What percent of the marks obtained in English approximately?

A: Marks obtained in English = 55⁰

$$360^0 = 100\%$$

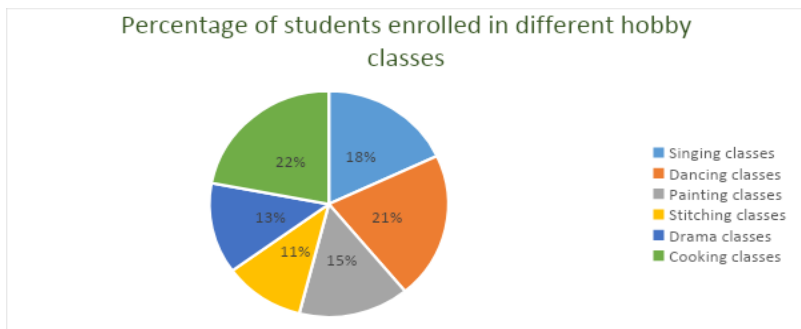
$$55^0 = x$$

By taking cross multiplication we get $360 \times x = 100 \times 55$

$$x = \frac{100 \times 55}{360} = \frac{275}{18} = 15.28\%$$

EXERCISE

1. Study the pie chart carefully to answer the questions that follow. Total number of students = 3600



QUESTIONS AND ANSWERS

1. The number of students enrolled in Cooking classes is what percentage of those enrolled in Dancing classes?

2. What is the total number of students enrolled in stitching and drama classes together?

3. How many students are enrolled in painting classes?

4. Number of students enrolled in painting classes is approximately what percent of those enrolled in singing classes?

5. What is the ratio of the number of students enrolled in singing and dancing classes together to those enrolled in drama classes respectively.

ANSWERS

1. 104.76%

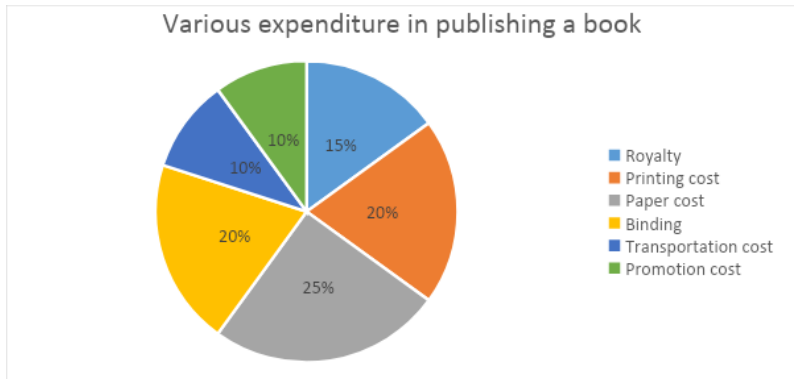
2. 864

3. 540

4. 83%

5. 3:1

2. Study the pie chart carefully to answer the questions that follow. Various expenditures incurred in publishing a book = Rs. 30,000



QUESTIONS AND ANSWERS

1. What is the expenditure spent on binding?
2. What is the difference between the expenditure spent on paper cost and royalty?
3. How much expenditure spent on printing cost?
4. How much expenditure is spent on transportation cost?
5. What is the ratio between the expenditures spent on promotion cost and to that of paper cost?

ANSWERS

1. 6000

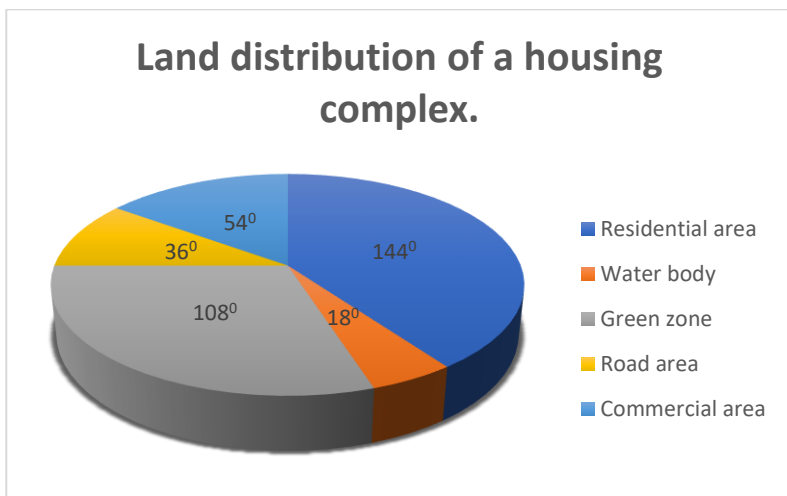
2. 3000

3. 6000

4. 3000

5. 2:5

3. The following pie chart shows the land distribution of a housing complex. If the total area of the complex is 5 acre, examine the pie chart and answer the questions.



QUESTIONS AND ANSWERS

1. What is the ratio of area allotted for residential to road purpose?

2. The percentage of total area allotted for the water body and green zone together is?

3. Land allotted green zone is greater than that for commercial purposes?

4. The total land allotted for residential and commercial purposes is?

ANSWERS

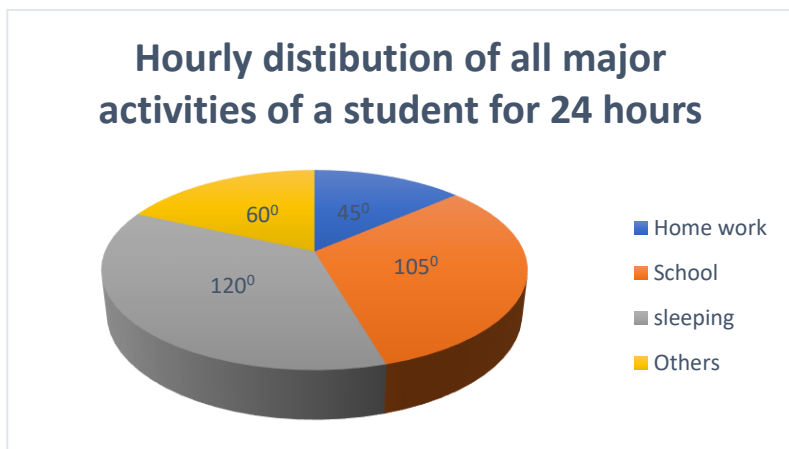
1.4:1

2. 35%

3. $\frac{3}{4}$ acre

4. $2\frac{3}{4}$ acre

4. The following pie chart shows the hourly distribution of all major activities of a student.



QUESTIONS AND ANSEWRS

1. The percentage of time, which he spends in school is

2. How much time (In percent) does he spend in games in comparison to sleeping?

3. If he spends the time in games equal to the homework and remains constant in other activities, then the percentage decreases in time of sleeping is?

4. What is the difference in time spent in school and in home work?

5. If he spends $\frac{1}{3}$ rd time of homework in mathematics, then the number of hours he spends in rest of the subjects in home work is?

ANSWERS

1. 30% 2. 25% 3. 12.5% 4. 4 hours 5. 2 hours

REFERENCES

1. https://en.wikipedia.org/wiki/Pie_chart
2. <https://youtu.be/Tbj6dTuZ8W8>
3. <https://www.youtube.com/watch?v=Vc2S7yLjyp0&t=14s>
3. <https://youtu.be/fA8cQW-nmIw>
4. <https://youtu.be/Vc2S7yLjyp0>



3.4. LINE GRAPHS

LEARNING OUTCOMES:

Upon practicing problems on Line graph students are able to:

1. visualize the changes in the quantities depending on base quantity.
2. compare the changes among different items over time through multiple line graphs.
3. predict the future trends of the items involved.

Introduction

Line graph is a data visualization tool that can be used to analyse the changes in one or more quantities with respect to the change in a base quantity. For example, change of temperature, wind speed over a period of time. The base quantity (like time) is called as independent variable and the quantities (temperature, wind speed) that are changing with respect to this independent variable are called dependent variables.

To draw a line graph, we take a point on the paper (plane) as reference point usually called as origin. The values of independent variable are represented on a horizontal line passing through the origin called as X-axis, while the dependent variable(s) values are identified on a vertical line through the origin known as Y-axis. The values of dependent variable(s) against a fixed value of independent variable are identified with points and set of all such points are connected by a line or some line segments which gives rise to a line graph. The X-axis and the Y-axis can be labelled according to the independent and dependent variables.

Generally, we deal with two types of line graphs

- Simple line graph
- Multiple line graph

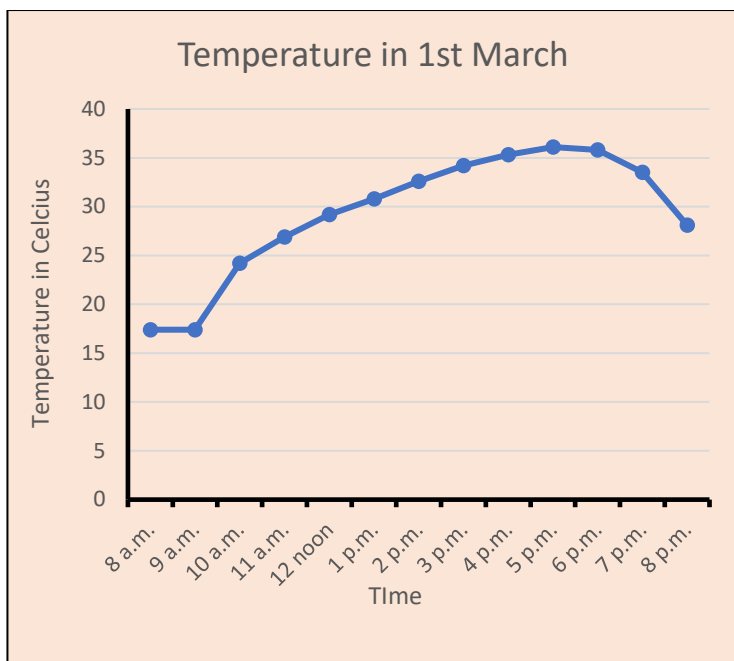


Figure 1: A simple line graph

A simple line graph depicts the changes in single dependent variable whereas multiple line graph is helpful to track the changes of more than one dependent variables. In multiple line graphs we can also compare the changes among the dependent variables involved.

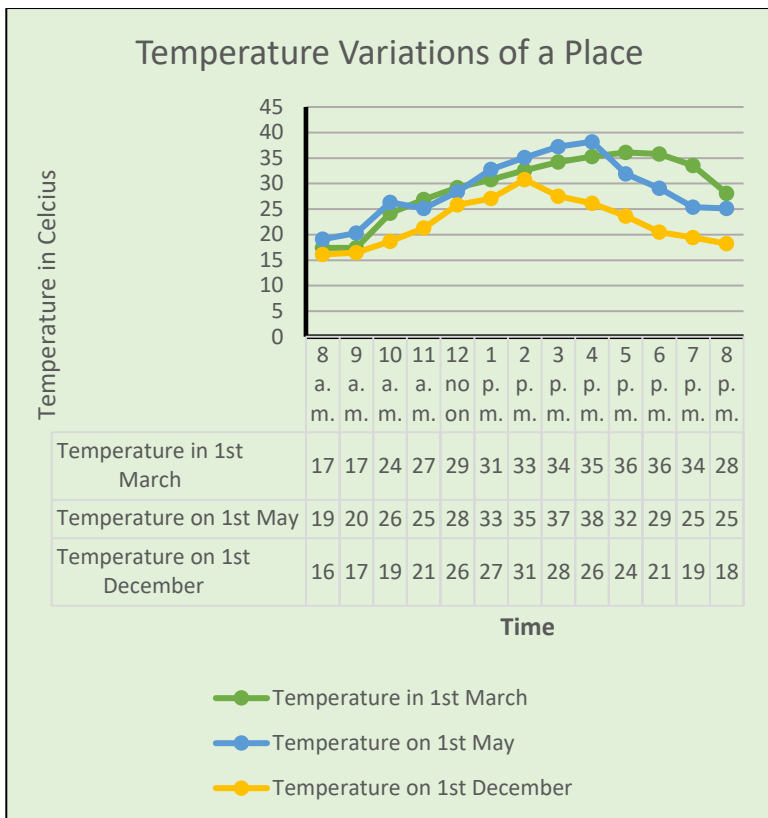


Figure 2: A multiple line graph

Example-1:

With reference to the above line graph (Figure 2), answer the following questions:

1. At what given time the difference between the temperatures of given place on 1st March and 1st December is maximum?
2. What is the difference between maximum temperatures of given place on 1st May and 1st December?
3. What percentage of change in temperature is noticed approximately from 8a.m to 8p.m on 1st May?
4. What is the ratio between the minimum and maximum temperatures on 1st May?
5. At what time the difference between the temperatures of given place on 1st March and 1st December is minimum?

Solution:

- 1) From the line graph we can easily notice that the gap between the graphs of 1st March and 1st December is wide between 5p.m. to 7p.m.

The difference of temperatures at 5p.m. = $36.1 - 23.6 = 12.5$

The difference of temperatures at 6p.m. = $35.8 - 20.5 = 15.3$

The difference of temperatures at 7p.m. = $33.5 - 19.4 = 14.1$

Therefore the difference in temperatures of 1st March and 1st December is maximum at 6p.m.

- 2) Difference between maximum temperatures on 1st May and 1st December

$$= 36.1 - 30.8 = 5.3^{\circ}\text{C}$$

- 3) Percentage of change in temperature from 8a.m to 8p.m on 1st May

$$= \frac{(25.1 - 19.1)}{19.1} \times 100 = 31.41\%$$

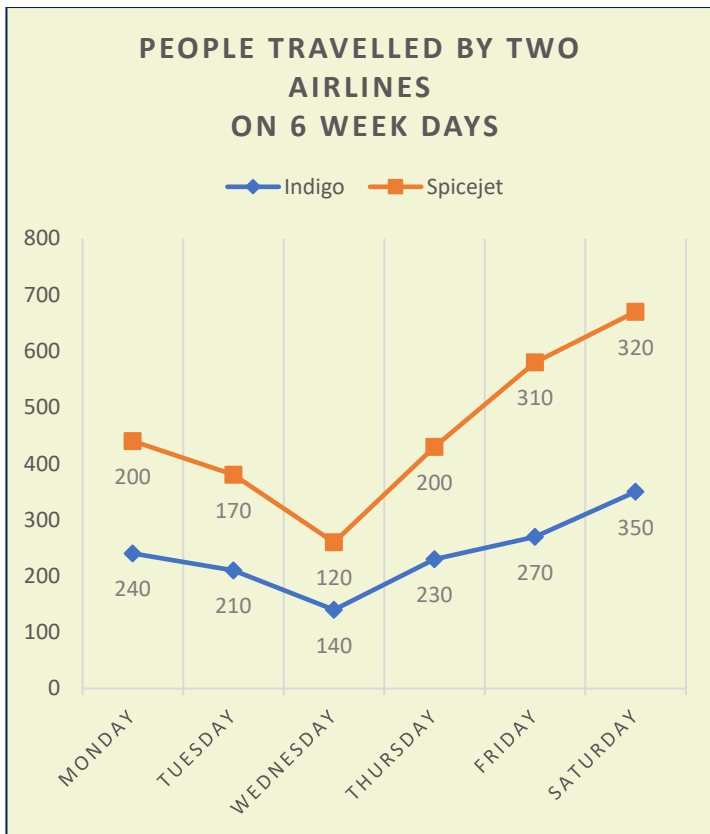
- 4) The ratio between the minimum and maximum temperatures on 1st May

$$= 19.1 : 38.2 = \frac{19.1}{38.2} = \frac{191}{382} = \frac{1}{2} = 1 : 2$$

- 5) By the line graph, we can observe that the difference between the temperatures of given place on 1st March and 1st December is minimum at 9a.m.

$$= 17.4 - 16.5 = 0.9^{\circ}\text{C}$$

Example-2: Refer to the following graph and answer the given questions.



- 1) What is the difference between the total number of people travelled by Indigo on Monday and Tuesday

together and the total number of people who travelled by SpiceJet on Friday and Saturday together?

- 2) What is the average number of people travelling by Indigo airlines on Monday, Tuesday, Wednesday and Thursday?
- 3) The number of people who travelled by Indigo airlines is increased by what percent from Wednesday to Saturday?
- 4) The total number of people who travelled by both the airlines together on Saturday is approximately what percent more than the total number of people who travelled by both the given airlines together on Wednesday?
- 5) What is the ratio of the people travelled by Indigo on Thursday and Friday put together to the people travelled by SpiceJet on Monday and Tuesday put together?

Solution:

- 1) Number of people travelled by Indigo on Monday and Tuesday = $240 + 210 = 450$

Number of people travelled by SpiceJet on Friday and

$$\text{Saturday} = 310 + 320 = 630$$

$$\text{The difference} = 630 - 450 = 180$$

- 2) Total number of people travelled by Indigo from Monday to Thursday

$$= 240 + 210 + 140 + 230 = 820$$

Average number of people travelled by Indigo from Monday to Thursday

$$= \frac{820}{4} = 205$$

- 3) The number of people who travelled by Indigo airlines is increased by what percent from Wednesday to Saturday?

The number of people travelled by Indigo on Wednesday = 140

The number of people travelled by Indigo on Saturday = 350

The number of people increased from Wednesday to Saturday = $350 - 140 = 210$

The percentage of increase = $\frac{210}{140} \times 100 = \frac{3}{2} \times 100 = 150\%$

- 4) The total number of people travelled by both the airlines together on Saturday

$$= 350 + 320 = 670$$

The total number of people travelled by both the airlines together on Wednesday

$$= 140 + 120 = 260$$

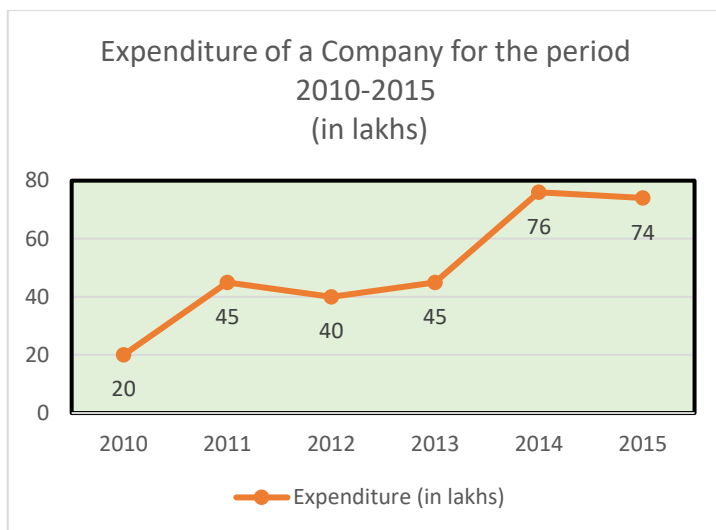
More number of people travelled from Wednesday to Saturday = $670 - 260 = 410$

$$\begin{aligned}\text{Required percentage of people} &= \frac{410}{260} \times 100 = \frac{41}{26} \times 100 = \\ 157.69\% &= 157\% \text{ approximately}\end{aligned}$$

- 5) The ratio of the people travelled by Indigo on Thursday and Friday put together to the people travelled by SpiceJet on Monday and Tuesday put together = $500 : 470$

$$= 50 : 47$$

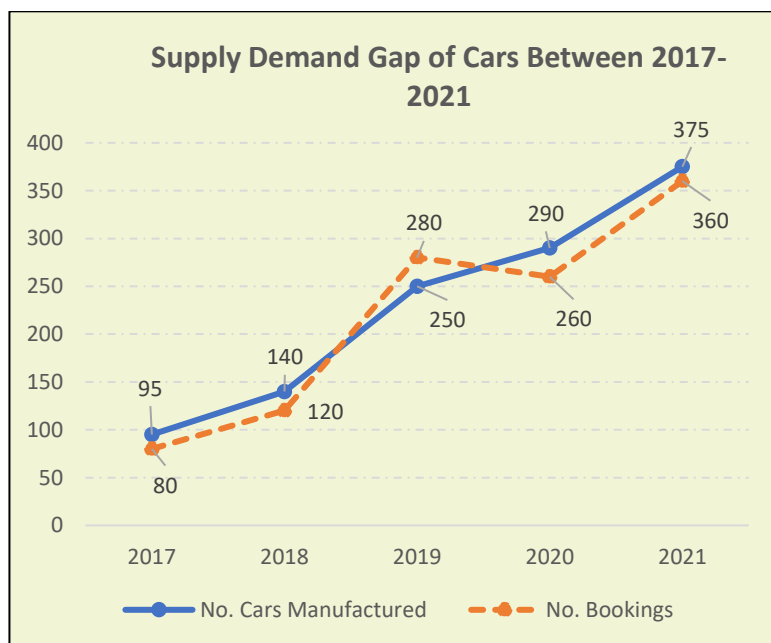
Problem-1: Study the following graph and answer the questions given below.



- 1) If the income of the company in the year 2010 was Rs. 50 lakhs and the income of the company in the year 2013 was Rs. 95 lakhs find the ratio of the profits of the company in the year 2010 to the expenditure of the company in the year 2013?
- 2) What is the approximate percent of profit of the year 2014, if the income of the company was Rs. 136 lakhs?
- 3) The profit of the company in the year 2011 was RS. 15 lakhs, what was the income of the company in that year?
- 4) In which year the percentage of increase/decrease in expenditure the highest from that of the previous year?

5) What is the average expenditure of the company over the given six years?

Problem-2: Study the following line graph and answer the given questions.



- 1) In which year the demand of cars is more than the supply?
- 2) In 2018, The production of cars is what percentage of demand?

- 3) The percentage increase of bookings is maximum in which year?
- 4) What is the difference between percentage of increase in manufacturing and bookings over the given period of time?
- 5) In which year the percentage of demand supply gap is highest?

Answers:

Problem-1: 1-3 : 5, 2-79%, 3-Rs.60 lakhs, 4-2011, 5-Rs.50 lakhs

Problem-2: 1-2019, 2-116.66%, 3-2019, 4-5.27%, 5-2020

Web Resources:

<https://bankexamtips.in/line-graph-data-interpretation-questions-and-answers/>

<http://hitbullseye.com/Line-Graph-Examples.php>



3.5 VENN DIAGRAMS

LEARNING OUTCOMES:

Upon practicing the problems on Venn diagrams students are able to:

1. visualize the interrelation among given sets .
2. analyses the interrelation among the different items.
3. based on interrelation among various items involved and draw valid conclusions.

Introduction

Venn diagram is a simple tool to visually represent inter-relationship among different groups of objects (commonly known as ‘Sets’). Each group of objects (Set) is represented by a simple closed curve like circle, ellipse, rectangle, triangle etc. The closed curves are to be drawn with or with out overlapping depending on the commonalities/inter-relationship among the sets represented by them. We will discuss how to represent some common cases of inter-relationship among different sets through Venn diagrams.

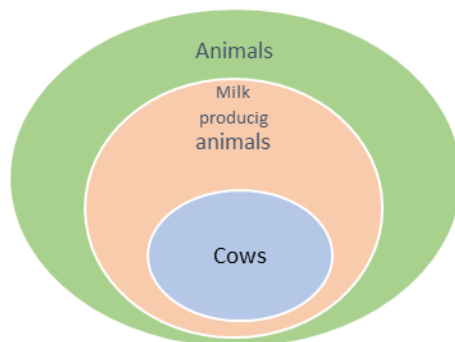
Case-1: All the given group of objects (Sets) are not related to each other. Such groups (sets) are called distinct groups (sets)

Example: Fruits, Animals, Birds



Case-2: Given group of objects (sets) are such that all the objects (elements) of first group (set) are also the objects of second group, all the objects of second group are also the objects of third group and so on. In this case, we say that first group is a sub-set of the second, the second is a sub-set of the third and so on. The third set is called a super-set of the first and second sets.

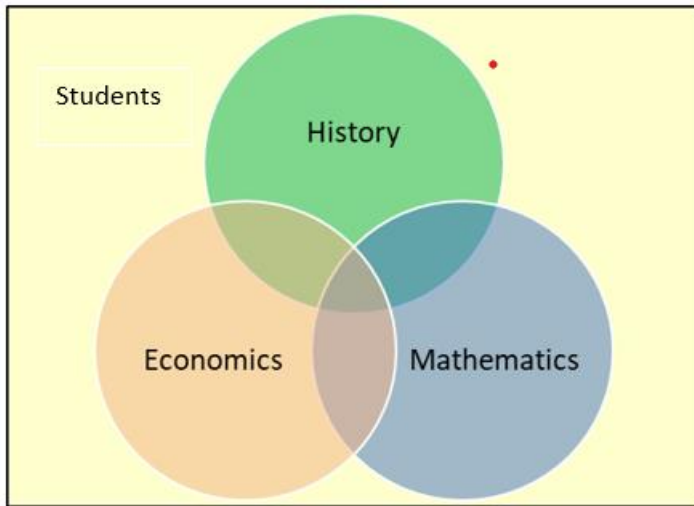
Example: Animals, Milk-producing animals, Cows



Case-3: Two or more group of objects (sets) are such that each group has some objects in common with one or more other groups but all of them are part of a bigger group (sub-sets of a bigger set). In a Venn diagram this bigger set (Universal set) is usually represented by a rectangle covering all the sets under discussion. All the groups of objects (sets) under discussion are sub-sets of the Universal set (bigger group)

Example: Doctors, Professors and Educated people

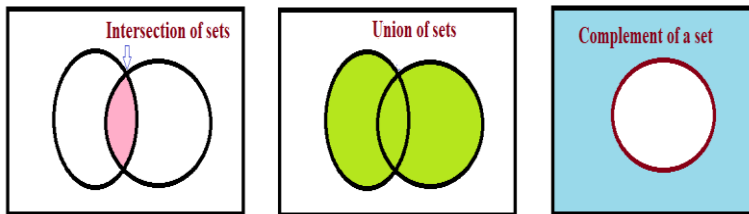
Example: History students, Economics students, Mathematics students and Students



Note that, in the above Venn diagram, the overlapping areas represent common objects of different groups (sets). And the area within the rectangle (representing Universal set) that is not covered by any one of the sets represents the objects not belongs to these individual sets but still part of bigger set.

It is also possible that within the Universal set (super-set of all the sets) the individual sets may have inter-relationships discussed in Case-1 and Case-2.

Finally, in a Venn diagram the overlapping area of two or more sets is called their intersection. The combined area of the sets (shaded part) is called their union and the area outside a given set but within the universal set is called the complement of the individual set. The intersection of two or more sets represents the common objects of those sets, the union of two or more sets represents the combined objects of all the sets put together and the complement of a set represents the objects which do not belong to that set (shaded part) but are part of universal set.



Example-1: Draw suitable Venn diagrams to represent the following group of objects (sets)

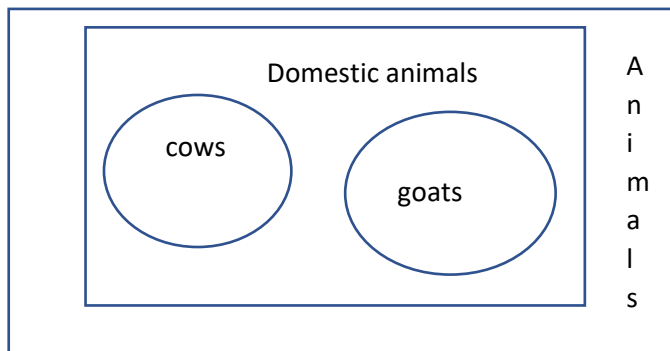
- A) Animals, Domestic animals, Goats, Cows
- B) Chairs, Tables, Wooden furniture
- C) Parents, Fathers, Brothers, Women

D) States, Districts, Union Territories

E) Food, Vegetables, Meat, Goat

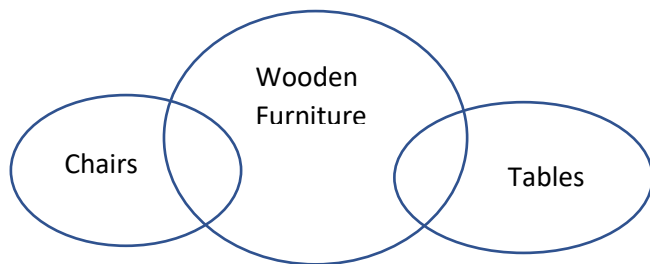
Solution:

A) We are given four group of objects (sets). Out of them all the Domestic animals are part of Animal group. Similarly, both Goats and Cows are Domestic animals but they are distinct groups (sets without common elements). Hence the Venn diagram to represent the given sets is as follows



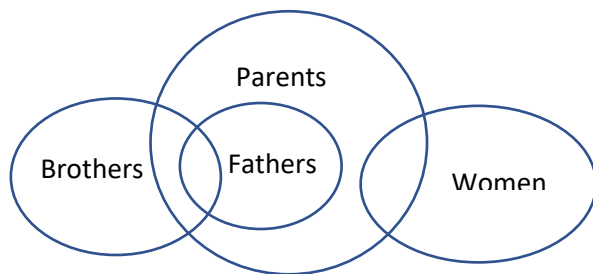
B) Chairs and Tables are distinct sets. Chairs and Tables are part of the set “Furniture” but they are not necessarily “Wooden Furniture”. Only some of the

chairs and some of the tables may be wooden. Thus, given three sets can be represented by the following Venn Diagram.



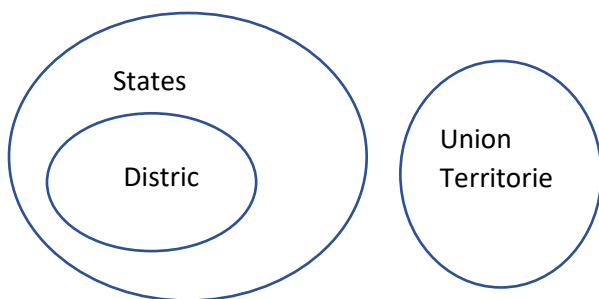
C) Parents, Fathers, Brothers, Women

Notice that every father is a parent. Some of the Brothers and some among the women can be Parents. Moreover, some Fathers can be Brothers and vice versa. But no women can be a Father or a Brother. So the Venn diagram representing the given sets is:



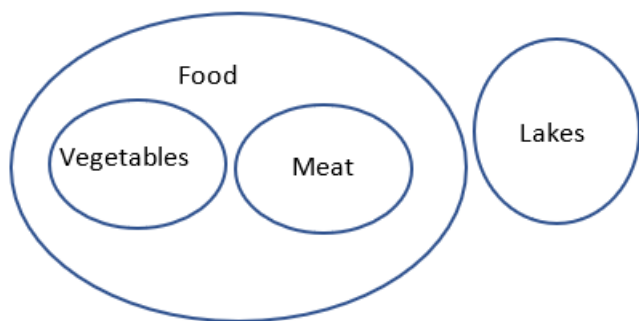
D) States, Districts, Union Territories

We know that every District is a part of States. But Union Territories are different administrative units in India. Thus the Venn diagram of given sets is as follows



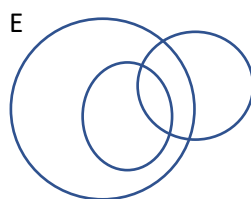
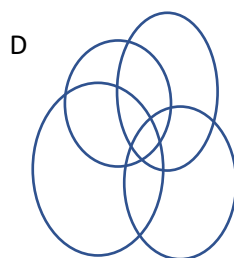
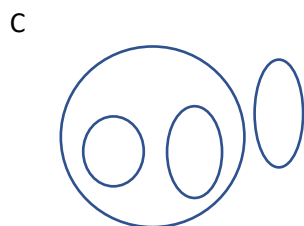
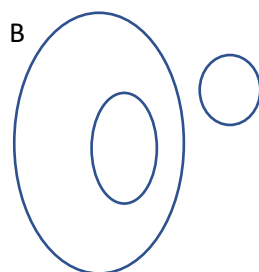
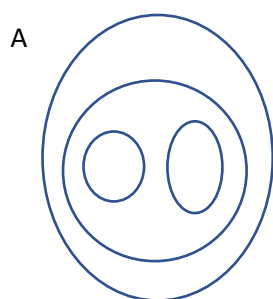
E) Food, Vegetables, Meat, Lakes

Vegetables and Meat are distinct groups of Food and Lakes is a water body that do not have any relation with food. Therefore, we can draw a Venn diagram for above groups as follows:



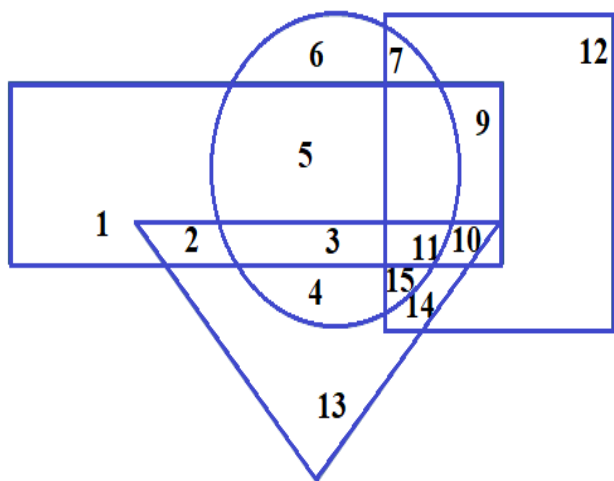
Problem-1: Examine the relationship among following sets carefully and match them with the Venn Diagrams given below.

- 1) Singer, Writer, Actor, Director
- 2) Police, Pick-pocket, Criminal
- 3) Mothers, Women, Advocates
- 4) Mammals, Humans, Men, Women
- 5) Minerals, Iron, Coal, Cotton



Answers: 1 – D, 2 – B, 3 – E, 4 – A, 5 – C

Example-2: In the Venn given below, circle represents employees, square represents married persons, triangle represents women and rectangle represents educated persons. Each section is numbered. Study the diagram to answer the following questions.



- 1) Which numbered region represents unmarried men are educated employees?

- 2) What section of people are represented by the region numbered 7?
- 3) Which numbered regions corresponds to unemployed married men?
- 4) What section of people are represented by the region numbered 14?

Solution:

- 1) Educated employees among men are represented by the regions where circle, rectangle and triangle meet. But if we want unmarried persons the region should be free from square. Therefore, our required region is where the circle, rectangle and triangle meet but free from square. That is region numbered '3'.
- 2) Region number 7 is where circle and square meets. Therefore '7' corresponds to Married employees.
- 3) Married men should be in the common region of square and triangle. But we want unemployed married men. So, the region should not be covered by circle. Hence, the regions numbered 10 and 14 represents unemployed married men.

- 4) Region number 14 is where triangle and square meets but free from rectangle and circle.

Hence, region 14 represents unemployed and uneducated married men.

Web Resources:

<https://mba.hitbullseye.com/Quant/Venn-Diagram-Problems.php>

<https://reasoningquestions.in/venn-diagram-questions-and-answers/>



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